Test Goodwin's Model Parameters Estimation and References

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1 Introduction

Goodwin's Model(1967, 1972) was simply a dynamical model of employment and workers' shares of output. Putted into a mathematical form, these two variables formed a pair of first order differential equations, which is similar to Lotka-Volterra system. With the following definitions and assumptions:

$$a = a_0 e^{\alpha t} \tag{1}$$
$$N = N_0 e^{\beta t} \tag{2}$$
$$\nu = \frac{K}{Y} \tag{3}$$
$$L = \frac{Y}{a} \tag{4}$$
$$\omega = \frac{w}{a} \tag{5}$$
$$\lambda = \frac{L}{N} \tag{6}$$
$$\dot{K} = (1-\omega)Y - \delta K \tag{7}$$
$$\frac{\dot{w}}{w} = \Phi(\lambda) = \phi_0 + \frac{\phi_1}{(1-\lambda)^2} \tag{8}$$

where α is productivity, N is population(total labour force), ν is capital-output ratio, L is labor force, K is capital stocks and Y is total output(GDP), ω is wage share, w is wage rate, λ is employment rate, δ is depreciation rate, and $\Phi(\lambda)$ is the nonlinear Phillips Curve.

Then the two dimensional ODEs system is

$$\begin{cases} \frac{\dot{\omega}}{\omega} = \phi_0 + \frac{\phi_1}{(1-\lambda)^2} - \alpha \\ \\ \frac{\dot{\lambda}}{\lambda} = \frac{1-\omega}{\nu} - \delta - \alpha - \beta \end{cases} \tag{9}$$

Interestingly, without further restricting the specific values for these parameters, we got the solution which is a family of closed, elliptical cycles with an unstable equilibrium at the center. The theoretical mechanics of the model gave us some reasonable insights about the conflicts between workers and capitalists. However, we still need to assess if Goodwin's result is plausible in the real world cases. What we will present in this report is to estimate the parameters $\alpha, \beta, \nu, \delta, \phi_0$ and ϕ_1 based on Harvie's methods and resources(2000) , which collected data from 10 OECD countries over 1970-2009 and compared the model set by the parameters with the real relation between employment and wage share in each of these countries.

2 Method of Estimation

The actual trajectory of the two variables of each country can be compared to the trajectory predicted by Goodwin's Model determined by the six parameters. On the one hand, since the cycle in the model's solution is perfect ellipse, its center $(\omega^\star, \lambda^\star)$ has the mean values of the two variables. Specifically, the econometric estimates $\hat{\alpha}, \hat{\beta}, \hat{\delta}, \hat{\phi_0}, \hat{\phi_1}$ can be used to calculate the 'econometric estimates', i.e.$(\hat{\omega^\star}, \hat{\lambda^\star})$. On the other hand, the center of the real trajectory, $(\bar{\omega}, \bar{\lambda})$, which has the mean values of the two variables, can be considered as the 'data estimates' of the center. Therefore, we have $(\hat{\omega^\star}, \hat{\lambda^\star}) = (\bar{\omega}, \bar{\lambda})$, and we get the following formulas:

$$\begin{cases} \bar{\omega} = \hat{\omega^\star} = 1 - (\hat{\delta} + \hat{\alpha} + \hat{\beta})\hat{\nu} \\ \\ \bar{\lambda} = \hat{\lambda^\star} = 1 - \sqrt{\frac{\hat{\phi_1}}{\hat{\alpha} - \hat{\phi_0}}} \end{cases} \tag{10}$$

Thus the model can be evaluated by comparing the 'econometric estimates' and 'data estimates'. At the same time we plot the actual values for our two variables, in addition to plotting these series in ω-λ plane, we can also plot the 'econometric estimates' of $(\omega^\star, \lambda^\star)$ on the plan.

2.1 Empirical Data

All the data are taken from the OECD countries from 1970-2009, and the data are annul, which means $\triangle t = 1$. Since we are looking for the real values of wage share and employment each year, we collected data of wage share(also known as labour income share, ω), population(total labour force, N), unemployment(UE), GDP(total output, Y) and capital-output ratio(ν) for each country. Then we can calculate the data for employment rate(λ), productivity(α) by following formulas:

$$\lambda = \frac{N - UE}{N} \tag{11}$$

$$\alpha = \frac{Y}{N - UE} \tag{12}$$

2.2 Parameter estimation

2.2.1 Productivity growth, α

From equation(1), to estimate α, we taking logs on both sides of the equation and add a stochastic error term:

$$ln(\alpha_t) = ln(\hat{\alpha_0}) + \hat{\alpha} t + \epsilon_{1t} \tag{13}$$

where t is the time trend. Table 1 shows the estimation results $\hat{\alpha}$ by performing linear regression on equation(13).

Table 1: Estimation Results for Productivity Growth

	Australia	Canada	Denmark	Finland	France	Germany	Italy	Norway	UK	US
Constant	8.623292‡	9.137655‡	7.81916‡	7.743135‡	9.965317‡	10.1141‡	9.965025‡	7.427985‡	10.13577‡	11.40713‡
	(0.005291)	(0.01553)	(0.010464)	(0.010288)	(0.002515)	(0.023558)	(0.009857)	(0.011105)	(0.005451)	(0.009914)
Time Trend	0.018223‡	0.018047‡	0.004772‡	0.004154‡	0.007684‡	0.014507‡	0.004111‡	0.01081‡	0.004756‡	0.014985‡
	(0.000225)	(0.00066)	(0.000445)	(0.000437)	(0.000107)	(0.001001)	(0.000419)	(0.000472)	(0.000232)	(0.000421)
R^2	0.994604	0.954512	0.763712	0.716953	0.993151	0.854917	0.729887	0.936404	0.922063	0.972606
$\bar{R}^2$	0.994462	0.953315	0.757494	0.709504	0.99297	0.851099	0.722779	0.934731	0.920012	0.971885
DW	0.221907‡	0.056552‡	0.122861‡	0.132911‡	0.533783‡	40.330637‡	0.159435‡	40.225664‡	0.240418‡	0.057507‡
Kstata	0.115018	0.136932	0.125409	0.098549	0.126043	0.118712	0.17899	0.083418	0.092261	0.140195
ARCH	21.37303‡	34.66311‡	28.30965‡	29.15474‡	17.9033‡	27.23471‡	30.47741‡	18.62343‡	19.75706‡	30.43618‡

Standard errors of estimates are in parentheses.
Note:‡ indicates parameter/test statistics is significant at the 5% level. A significant test statistics means the specification fails the test, i.e., the null hypothesis of no serial correlation or correct functional form, etc. is rejected.

2.2.2 Population growth, β

Similarly, for the growth rate of population β, we take logs of equation(2) and adding an error term and obtain:

$$ln(N_t) = ln(\hat{N}_0) + \hat{\beta}t + \epsilon_{2t} \tag{14}$$

Table 2 shows the estimation results for β.

Table 2: Estimation Results for Population Growth

	Australia	Canada	Denmark	Finland	France	Germany	Italy	Norway	UK	US
Constant	8.623292‡	9.137655‡	7.81916‡	7.743135‡	9.965317‡	10.1141‡	9.965025‡	7.427985‡	10.13577‡	11.40713‡
	(0.005291)	(0.01553)	(0.010464)	(0.010288)	(0.002515)	(0.023558)	(0.009857)	(0.011105)	(0.005451)	(0.009914)
Time Trend	0.018223‡	0.018047‡	0.004772‡	0.004154‡	0.007684‡	0.014507	0.004111‡	0.01081‡	0.004756‡	0.014985‡
	(0.000225)	(0.00066)	(0.000445)	(0.000437)	(0.000107)	(0.001001)	(0.000419)	(0.000472)	(0.000232)	(0.000421)
R^2	0.994604	0.954512	0.763712	0.716953	0.993151	0.854917	0.729887	0.936404	0.922063	0.972606
$\bar{R}^2$	0.994462	0.953315	0.757494	0.709504	0.99297	0.851099	0.722779	0.934731	0.920012	0.971885
DW	0.221907‡	0.056552‡	0.122861‡	0.132911‡	0.533783‡	0.330637‡	0.159435‡	0.225664‡	0.240418‡	0.057507‡
Kstata	0.115018	0.136932	0.125409	0.098549	0.126043	0.118712	0.17899	0.083418	0.092261	0.140195
ARCH	21.37303‡	34.66311‡	28.30965‡	29.15474‡	17.9033‡	27.23471‡	30.47741‡	18.62343‡	19.75706‡	30.43618‡

Standard errors of estimates are in parentheses.
Note:‡ indicates parameter/test statistics is significant at the 5% level. A significant test statistics means the specification fails the test, i.e., the null hypothesis of no serial correlation or correct functional form, etc. is rejected.

2.2.3 Capital-output ratio, ν

As Goodwin assumes a constant capital-output ratio, so we calculate the estimate of ν of each country by calculating its mean. Table 3 shows the estimation results for ν.

Table 3: Estimation Results for Capital-Output Ratio

	Australia	Canada	Denmark	Finland	France	Germany	Italy	Norway	UK	US
ν	2.996291	2.427188	3.498316	3.627691	2.984397	3.710201	3.421321	3.800333	2.585883	2.523322

2.2.4 Phillips curve and depreciation, ϕ and δ

Harvie didn't consider the effect of capital depreciation and to estimate the coefficients in Phillips Curve, he used a discretization method of continuous model which could be summarized as the followings. Since the Goodwin Model is a continuous model and the economic data are discrete, we replace the differential equation with its difference equation analogue

$$\frac{\omega(t+1)-\omega(t)}{\omega(t)} = \phi_0 - \alpha + \frac{\phi_1}{(1-\lambda(t))^2} \tag{15}$$

Then to get the coefficients, which is the long-run relationship between real-wage growth and employment rate, and to offset the errors come out when we change the differential equation into difference equation, we estimate a dynamic formulation including lags of both dependent and independent variables, from which long-run coefficients can be extracted. let $\omega^{'}(t) = \frac{\omega(t+1)-\omega(t)}{\omega(t)}$ and $\tilde{\lambda}(t) = \frac{1}{(1-\lambda(t))^2}$, then the econometric equation to be estimated will be

$$\omega^{'}(t) = \hat{\phi}_{00} + \sum_{k=1}^{m} \hat{\phi}_{0k}\omega^{'}(t-k) + \sum_{k=0}^{n} \hat{\phi}_{1k}\tilde{\lambda}(t-k) \tag{16}$$

where m and n are the number of lags necessary to ensure the model is dynamically well specified.

After trying with different values for m and n to perform linear regression on equation(16), we adopt the values of m and n which yield the largest $\bar{R}^2$ for the regression and then we take the estimated value of ϕ_0 and ϕ_1 as:

$$-\hat{\alpha} + \hat{\phi}_0 = \frac{\hat{\phi}_{00}}{1 - \sum\limits_{k=1}^{m} \hat{\phi}_{0k}} \qquad \hat{\phi}_1 = \frac{\sum\limits_{k=0}^{m} \hat{\phi}_{1k}}{1 - \sum\limits_{k=1}^{m} \hat{\phi}_{0k}} \tag{17}$$

However, the above method is kind of subjective and difficult-implemented in choosing m and n, and the regression results, with the largest $\bar{R}^2$ also below 0.2, shows that such method is also inprecise. What's more, since Harvie has not considered the value of δ, which is also difficult to obtain. Therefore, the value of δ is usually simply and randomly chosen as a figure between 3% and 8% as is used in a lot of economics papers.

Here we present another way to estimate the value of ϕ_0, ϕ_1 and δ by using the first integral of the method. From the dynamics of the Goodwin Model in equation (9), we can get

$$\frac{\dot{\omega}}{\omega}\left(\frac{1-\omega}{\nu}-\alpha-\beta-\delta\right)=\frac{\dot{\lambda}}{\lambda}\left(\phi_0-\alpha+\frac{\phi_1}{(1-\lambda)^2}\right) \tag{18}$$

Integrate both sides of the above equation, we have

$$\begin{aligned} & \left(\tfrac{1}{\nu}-\alpha-\beta\right)\log\omega-\tfrac{\omega}{\nu}-\delta\log\omega \\ = \; & (\phi_0-\alpha)\log\lambda+\phi_1\left(\log\lambda-\log(1-\lambda)+\tfrac{1}{1-\lambda}\right)+C \end{aligned} \tag{19}$$

which is a first integral of the system. Then if we take $\tilde{\omega}$ as $(\frac{1}{\nu}-\alpha-\beta)\log\omega-\frac{\omega}{\nu}$, ω_1 as $\log\omega$, λ_1 as $\log\lambda$, and λ_2 as $\log\lambda-\log(1-\lambda)+\frac{1}{1-\lambda}$, we get

$$\tilde{\omega}=C+(\phi_0-\alpha)\lambda_1+\phi_1\lambda_2+\delta\omega_1 \tag{20}$$

At this time, by using high-dimensional linear regression, we can get the estimated value for δ, ϕ_1 and ϕ_0, which are shown in Table 4.

Table 4: Estimation Results for δ, ϕ_0 and ϕ_1

	Australia	Canada	Denmark	Finland	France	Germany	Italy	Norway	UK	US
Constant	−0.324641‡	−0.318196‡	−0.321676‡	−0.323444‡	−0.323340‡	−0.313995‡	−0.323507‡	−0.298099‡	−0.308990‡	−0.310719‡
	(0.00092)	(0.00106)	(0.00021)	(0.00060)	(0.00047)	(0.00035)	(0.0023)	(0.00156)	(0.00036)	(0.00027)
ϕ_0	−0.022749‡	0.005900	-0.000236	-0.005506	−0.021704‡	−0.007428‡	−0.029208‡	0.038531	-0.000176	0.001493
	(0.00591)	(0.00527)	(0.00079)	(0.00287)	(0.00438)	(0.00242)	(0.01173)	(0.02061)	(0.00126)	(0.00182)
ϕ_1	2.30E-5	-6.37E-5	8.52E-7	1.02E-5	$5.3E-5$‡	2.74E-6	8.15E-5	-9.81E-6	6.83E-7	-4.17E-6
	(1.26E-05)	(0.44E-05)	(7.30E-07)	(8.90E-06)	(9.83E-06)	(1.44E-06)	(8.05E-05)	(1.23E-05)	(3.56E-06)	(7.14E-06)
δ	0.093218‡	0.096357‡	0.080300‡	0.0801576‡	0.067309‡	0.074561‡	0.066154‡	0.094948‡	0.069723‡	0.078825‡
	(0.00120)	(0.00062)	(0.00056)	(0.00122)	(0.00079)	(0.00136)	(0.00076)	(0.00200)	(0.00086)	(0.0004)
R^2	0.995303	0.998726	0.998385	0.994693	0.997308	0.994833	0.996032	0.989284	0.995702	0.999274
$\bar{R}^2$	0.995179	0.99869	0.998342	0.994553	0.997237	0.994697	0.995927	0.989002	0.995589	0.999255
DW	0.878758‡	0.849856‡	1.099926‡	0.981912‡	0.342031‡	0.421784‡	0.906306‡	0.686058‡	1.303388‡	0.968609418‡
Kstata	0.173348	0.114250	0.170017	0.16641	0.113044	0.139331	0.0924489	0.122234	0.2553898‡	0.185704
ARCH	3.505278	7.353806	2.775145	0.565145	18.71522	9.383174	3.905445	6.437244	0.004112	0.006990

Standard errors of estimates are in parentheses.
Note:‡ indicates parameter/test statistics is significant at the 5% level. A significant test statistics means the specification fails the test, i.e., the null hypothesis of no serial correlation or correct functional form, etc. is rejected.

3 Result

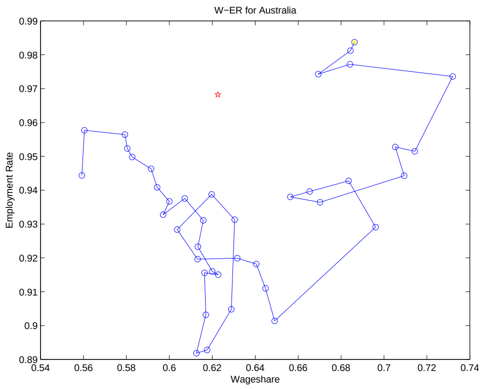
After getting the values of the two variables for each year, we can calculate the 'data estimates' $\bar{\omega}$, $\bar{\lambda}$. After estimating all the parameters, we present our 'econometric estimates' of the center, $(\hat{\omega^{\star}}, \hat{\lambda^{\star}})$ and the period for the ellipse cycle $\hat{T}$ for each country. Here we present all our estimateing results for the ten countries, including the 'econometric estimates' as well as our 'data estimates' in Table 5.

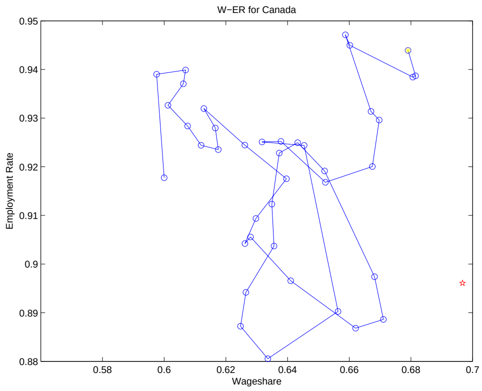
Now we present the ω-λ trajectory graphs of the ten countries. The yellow points notate the data of 1970 and the red pentagrams notate the 'econometric estimates' of the center.

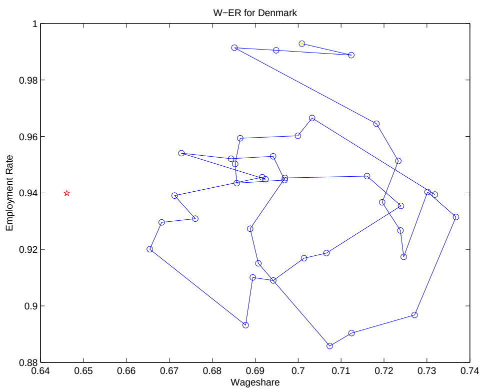
Table 5: Final Estimation Results for Goodwin Model

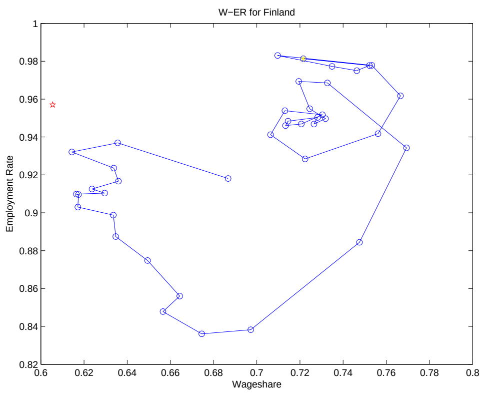
	Australia	Canada	Denmark	Finland	France	Germany	Italy	Norway	UK	US
$\hat{\alpha}$	0.0145	0.0105	0.0161	0.0244	0.0163	0.0086	0.0181	0.0214	0.0206	0.0149
$\hat{\beta}$	0.0182	0.0180	0.0048	0.0042	0.0077	0.0145	0.0041	0.0108	0.0048	0.0150
$\hat{\nu}$	2.996	2.427	3.498	3.628	2.984	3.710	3.421	3.800	2.586	2.523
$\hat{\phi}_0$	-8.23E-03	1.64E-02	1.59E-02	1.89E-02	-5.40E-03	1.22E-03	-1.11E-02	-1.72E-02	2.05E-02	1.64E-02
$\hat{\phi}_1$	2.30E-05	-6.37E-05	8.52E-07	1.02E-05	5.33E-05	2.74E-06	8.15E-05	-9.81E-06	6.83E-07	-4.17E-06
$\hat{\delta}$	0.0932	0.0964	0.0803	0.0802	0.0673	0.0746	0.0662	0.0949	0.0697	0.0788
$\hat{\omega^\star}$	0.623	0.697	0.646	0.605	0.728	0.637	0.698	0.517	0.754	0.726
$\bar{\omega}$	0.635	0.639	0.700	0.694	0.725	0.721	0.749	0.633	0.699	0.676
$\hat{\lambda^\star}$	0.968	0.896	0.940	0.957	0.950	0.981	0.947	0.000†	0.938	0.947
$\bar{\lambda}$	0.936	0.919	0.939	0.929	0.927	0.937	0.912	0.968	0.934	0.939
Period	0.0794	0.4151	0.2222	0.1400	0.1441	0.0407	0.1736	0.0000†	0.1864	0.1463

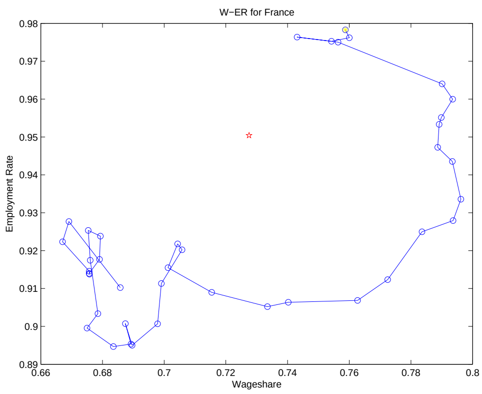
Note:† indicates that Goodwin Model cannot give a predicted value for the term according to parameter estimation results.

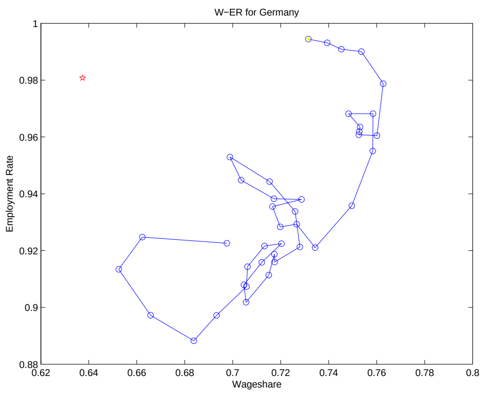


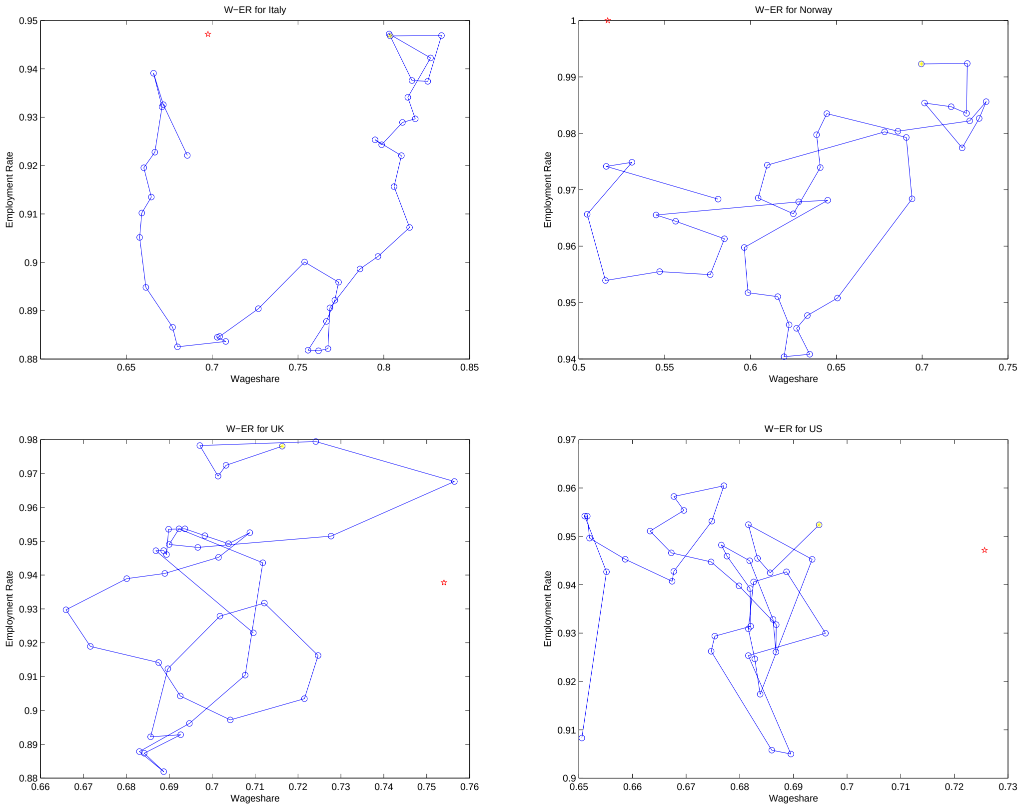












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