Some Discussions about
Investment Propensity Function

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Abstract

In this paper, we discuss some necessary conditions of this function, give some examples or suggested forms and show that different choices of this function could yield to totally different results in stability analysis of Keen Model.

1 Introduction

Investment propensity function, which is also called investment propensity curve, is used in Keen's model to indicate the investment propensity of the firms. The definition of this function is

$$\kappa(\pi) = \frac{I}{Y} \tag{1}$$

where Y means total yearly output or GDP and I means investment, which means the proporation ratio of the total output that are used in investment, and we assume that it is a continuous function of π, the profit share. Previously, researchers and economists have used different forms and formulas of this curve in their papers without giving their reason for their choices.

However, Close examination shows that with certain choices of the form, we can effectively constrain the wage share to $[0, 1]$, and get some other important characteristics of the model. For example, with some curves, we can get a stable equilibrium in the Keen's model, but with others, we can get a limit cycle, or an unstable fixed point. Therefore, choices and adoptions of the form and the formula of this curve might be significant and we should understand the influence of this curve to the whole system.

2 Necessary Conditions for $\kappa(\pi)$

Based on the stock-flow consistency table, or the accountant formula $Y = C + I$, where Y means total yearly output or GDP, C means total consumption and I means investment (We don't consider government and foreigh sector), we can get

$$\kappa(\pi) \leq 1 \tag{2}$$

based on the definition of $\kappa(\pi)$ and the fact that $C \geq 0$.

Additionally, based on the economy insight that high net profits lead to more borrowing whereas low net profit (possibly negative) lead to a deleveraging of the economy, we can get the point that

$$\kappa'(\pi) \geq 0 \tag{3}$$

when $\pi \in (-\infty, 1)$, Also, we can assume that $\kappa(\pi) > \pi$ when π is at a high level and conversely $\kappa(\pi) < \pi$ when π is at a low level.

Besides, we consider the critical value of π between $\kappa(\pi) > \pi$ and $\kappa(\pi) < \pi$. we define the capital interest rate as

$$r_c = \frac{\Pi - \delta K}{K} = \frac{\pi}{\nu} - \delta \tag{4}$$

and based on economic sense, when $r_c = 0$, in which firms make zero profit after paying the wages, loan interests and subtracting the depreciation of capital, they would neither borrow more debts from the banks nor paying off their debts in the bank, which means that they would just invest exactly the amount of profit into investment and thus $\kappa(\overline{\pi}) = \overline{\pi}$ where $\overline{\pi} = \nu\delta$. Therefore, we get the following equation

$$\kappa(\nu\delta) = \nu\delta. \tag{5}$$

To sum up, the investment propensity curve $\kappa(\pi)$ should satisfies (2),(3) and (5).

3 Examples and Suggested forms

3.1 Grasselli, 2011

$$\kappa_1(\pi) = -e^{-5} + e^{20\pi - 5} \tag{6}$$

The figure of this curve can been seen in Figure 1.

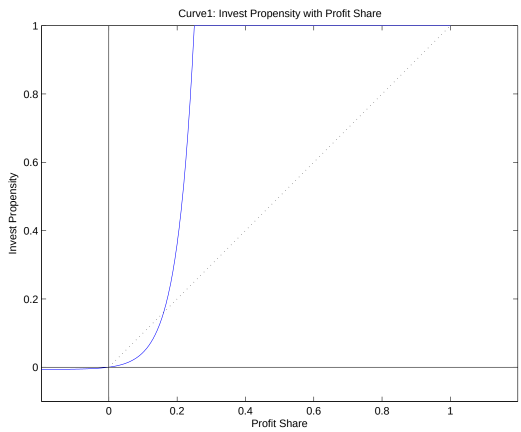

Figure 1: $\kappa_1(\pi)$

3.2 Desai, 2005

Desai assumed that there is a lower bound of profit rate (as well as profit share $\pi_{min}(\geq 0)$) and he assumed that as $\pi \to \pi_{min}^+, \kappa(\pi) \to -\infty$. Then a suggested form could be

$$\kappa_2(\pi) = Alog\frac{\pi - \pi_{min}}{\pi_{min}} \tag{7}$$

or

$$\kappa_2(\pi) = A + \frac{B}{\pi - \pi_{min}} \tag{8}$$

With this form, it's easy to proove that ω will be upper bounded by 100%. Then, when considering the restrictions of (2),(3) and (5), and we assume that $\nu = 3$ and $\delta = 6\%$, a specific formula could be

$$\kappa_2(\pi) = 1.18 + \frac{0.18}{\pi} \tag{9}$$

The figure of this curve can been seen in Figure 2.

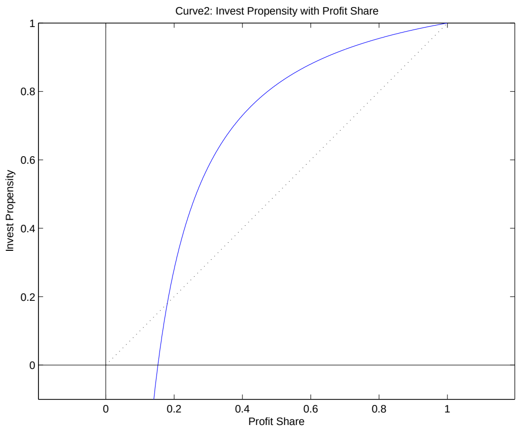

Figure 2: $\kappa_2(\pi)$

3.3 Lower Bounded κ

Borrowing the idea in $\kappa_2(\pi)$, we still assume that $\kappa(\pi)$ takes the form of $\kappa(\pi) = a + \frac{b}{\pi - c}$ whereas $c \leq 0$ to make $\kappa(\pi)$ lower bounded when $\pi \in (0, 1)$. Then when considering the restrictions of (2),(3) and (5), and we assume that $\nu = 3$ and $\delta = 6\%$, a specific formula could be

$$\kappa_3(\pi) = 2 - \frac{1.82}{\pi + 0.82} \tag{10}$$

The figure of this curve can been seen in Figure 3.

3.4 Non-negative κ

To avoid the situation which $\kappa(\pi) < 0$, we take

$$\kappa_4(\pi) = max\{\kappa_3(\pi), 0\}. \tag{11}$$

The figure of this curve can been seen in Figure 4.

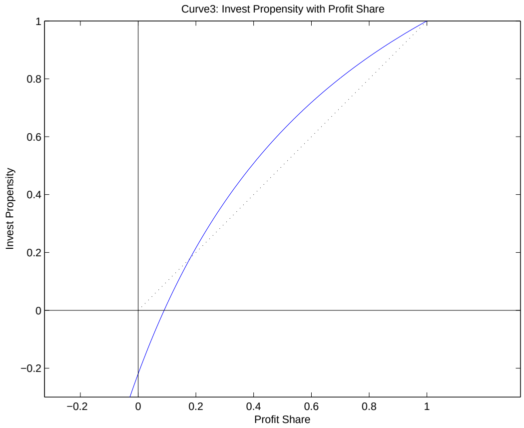

Figure 3: $\kappa_3(\pi)$

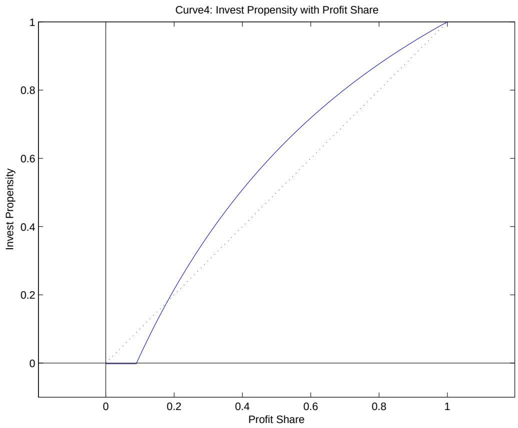

Figure 4: $\kappa_4(\pi)$

3.5 Sigmoid κ

We assume $\kappa(\pi)$ takes a sigmoid curve thus

$$\kappa(\pi) = \frac{1}{A + e^{-B(\pi - C)}} \tag{12}$$

and when considering the restrictions and we assume that $\kappa(\pi)$ also satisfies

$$\kappa(0) \approx 0 \tag{13}$$

then we get $A = 1, B = 20, C = 0.256$, so

$$\kappa_5(\pi) = \frac{1}{1 + e^{-20(\pi - 0.256)}} \tag{14}$$

The figure of this curve can been seen in Figure 5.

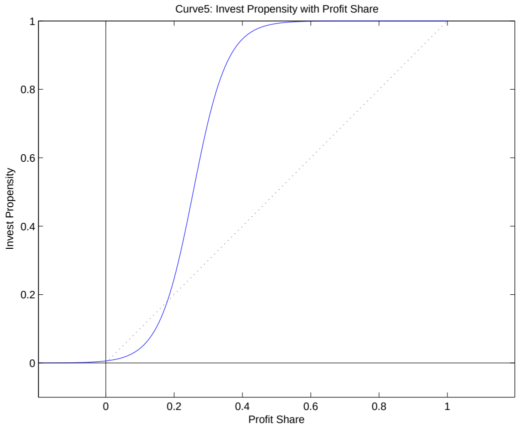

Figure 5: $\kappa_5(\pi)$

4 Further discussions

In this section, we want to show the differences it makes on Keen's model from choosing different formulas for the investment propensity curves.

The dynamics of Keen Model is formulated as follows:

$$\begin{cases} \dot{\omega} = \omega(\Phi(\lambda) - \alpha) \\ \dot{\lambda} = \lambda(\frac{\dot{K}}{K} - \alpha - \beta) \\ \dot{d} = -d \cdot \frac{\dot{K}}{K} + \kappa(\pi) - \pi \end{cases} \tag{15}$$

where ω, λ and d is the wage share, employment rate and debt ratio respectively, $\Phi(\lambda)$ is Phillips Curve, α, β and $\frac{\dot{K}}{K} = \frac{\kappa(\pi)}{\nu} - \delta$ is the growth rate of productivity, total labour force and capital stock respectively, with δ as capital depreciation rate. Also, in this case, $\pi = 1 - \omega - r \cdot d$ where r is the loan interest rate.

Now we choose the fundamental economic constants to be

$$\alpha = 0.015, \quad \beta = 0.01, \quad \delta = 0.06, \quad \nu = 3, \quad r = 0.03. \tag{16}$$

and take the Phillips Curve to be

$$\Phi(\lambda) = 0.005 + \frac{4.9 \times 10^{-5}}{(1-\lambda)^2}. \tag{17}$$

Also, we set the initial condition to be $(\omega_0, \lambda_0, d_0) = (0.75, 0.92, 0.5)$.

Then with the same parameters as well as initial values, simply by using different investment propensity curve from $\kappa_1(\pi)$ to $\kappa_5(\pi)$, it comes up with distinct simulation results which was shown from Figure 6 to 10.

As we can see, $\kappa_1(\pi)$ takes the trajectory to the 'bad equilibrium' $(0, 0, +\infty)$, $\kappa_2(\pi)$, $\kappa_3(\pi)$ and $\kappa_4(\pi)$ each creates a limit cycle and $\kappa_5(\pi)$ takes the trajectory to the 'good equilibrium' 1500 years later.

Further simulation with $\kappa_3(\pi)$ can even convince us that with this investment propensity

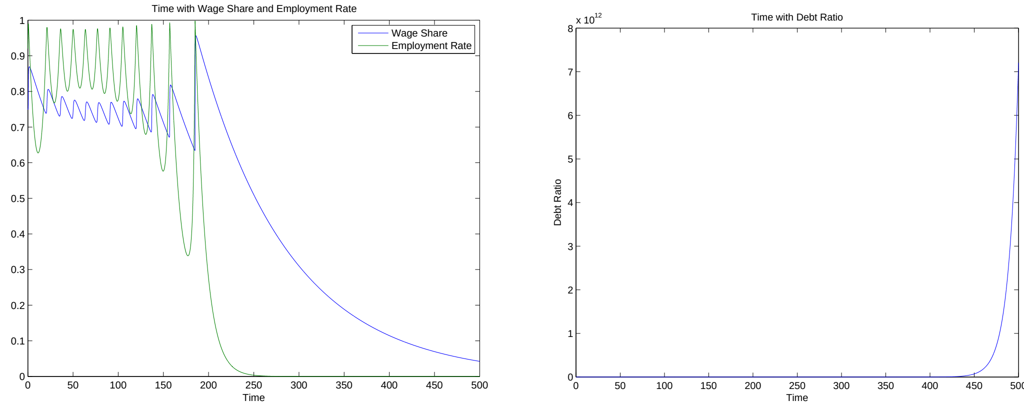

Figure 6: Simulation Results for $\kappa_1(\pi)$

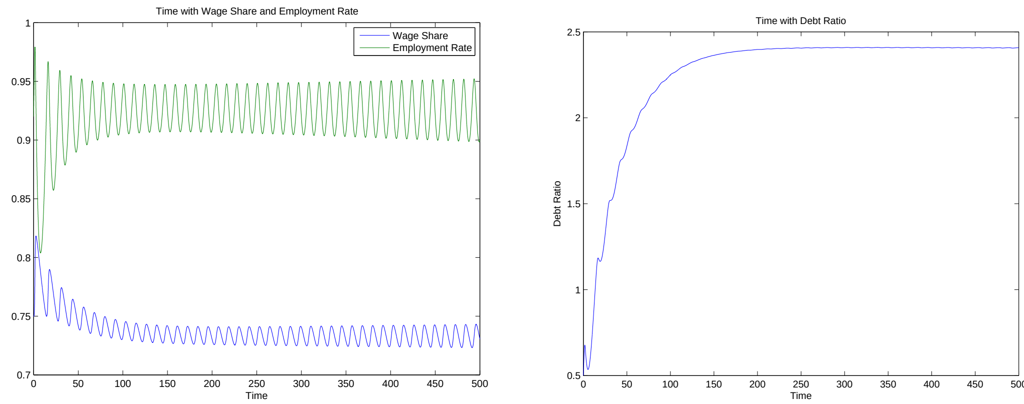

Figure 7: Simulation Results for $\kappa_2(\pi)$

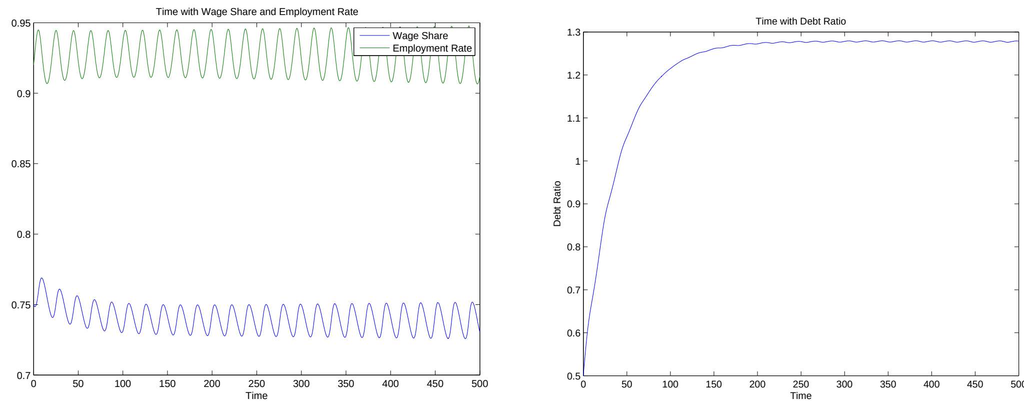

Figure 8: Simulation Results for $\kappa_3(\pi)$

curve, the Keen Model may not have the 'bad equilibrium'. For example, when take the initial value as $(\omega_0, \lambda_0, d_0) = (0.5, 0.6, 1)$ which is far from the normal state, if there is such a 'bad equilibrium', then the trajectory is believed to tend it. However, simulation shows that the trajectory also come to the limit cycle. This result is shown in Figure 11.

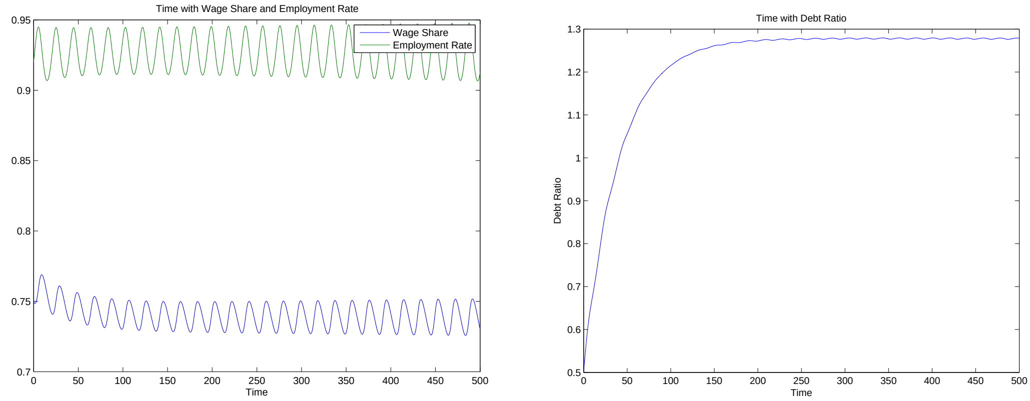

Figure 9: Simulation Results for $\kappa_4(\pi)$

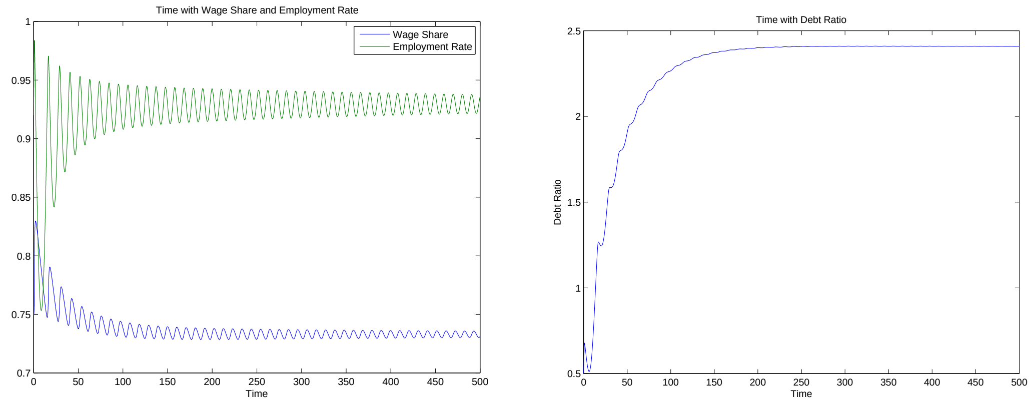

Figure 10: Simulation Results for $\kappa_5(\pi)$

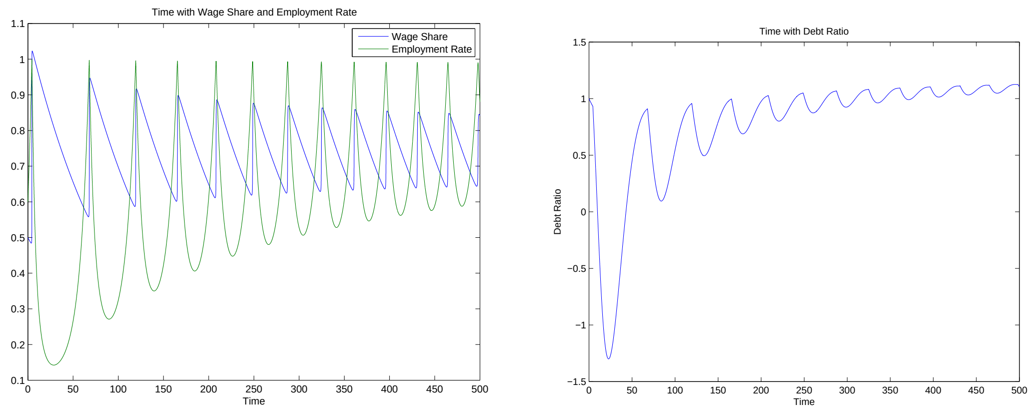

Figure 11: Simulation Results for $\kappa_3(\pi)$ with $(\omega_0, \lambda_0, d_0) = (0.5, 0.6, 1)$

5 Conclusion

From the discussion before, the investment propensity function is a very sensitive factor in the Keem Model and it can cause great difference and even become the dicisive factor about the model stability. Thus we should be careful when this function is used in economic modeling just like Keen model.

6 References

1. M. R. Grasselli, An analysis of the Keen model for credit expansion, asset price bubbles and financial fragility, Math Finan Econ, DOI 10.1007/s11579-012-0071-8
2. M. Desai, A clarification of the Goodwin model of the growth cycle, Journal of Economic Dynamics & Control, **30** (2006) 2661-2670