Some results on Non-Constant Capital Output Ratio

Hao Yin and Kimsy Tor
Fields Institute

September 11, 2013

In original Goodwin model, we assume that there is a constant capital output ratio. However, when we observe the real data of capital output ratio, the assumption does not hold. In this paper, we will assume a non constant capital output ratio and reexamine the original goodwin model.

Two diffrent theories on the non constant capital output ratio will be discussed as following:

1 Capital Utilization Theory by Keen and Desai

Not all of the capital is utilized all the time. We then assume C, a portion of capital involved in the reproduction. We,therefore, will obtain a constant or "true" capital output ratio, ν^*.

$$\frac{K \cdot C}{Y} = \nu^* \quad \text{constant} \tag{1}$$

where C is the Capital Utilization. According to the data from the real world,we assume that C is an increasing function of λ. C is now written as C(λ). There are three different forms of C(λ):

- (Desai) C(λ)=λ^Q
- (Keen) C(λ)=$\frac{1}{S+e^-Q*(\lambda-R)}$
- (Our own assumption) C(λ)=$Q + R \cdot \lambda$

where Q,R,S are constant. We derive the equation as following:

$$\begin{aligned}
\text{Note that} \qquad Y &= \frac{K * C}{\nu^*} \\
&= L \cdot a \\
&= N \cdot \lambda \cdot a. \\
\dot{K} &= Y \cdot (1-\omega) - \delta \cdot K \\
\text{Then} \qquad \frac{\dot{Y}}{Y} &= \frac{\dot{K}}{K} + \frac{\dot{C}}{C} \\
\Rightarrow \frac{\dot{\lambda}}{\lambda} + \frac{\dot{a}}{a} + \frac{\dot{N}}{N} &= \frac{Y \cdot (1-\omega) - \delta K}{K} + \frac{C'(\lambda)}{C(\lambda)}\dot{\lambda} \qquad (2) \\
\Rightarrow \frac{\dot{\lambda}}{\lambda} + \alpha + \beta &= \frac{C \cdot (1-\omega)}{\nu^*} + \frac{C'(\lambda)}{C(\lambda)}\dot{\lambda} - \delta \\
\Rightarrow \frac{\dot{\lambda}}{\lambda} - \frac{C'(\lambda)}{C(\lambda)}\dot{\lambda} &= \frac{C \cdot (1-\omega)}{\nu^*} - \alpha - \beta - \delta \\
\Rightarrow \frac{\dot{\lambda}}{\lambda} &= \frac{C \cdot (1-\omega) - \nu^* \cdot (\alpha + \beta + \delta)}{\nu^*(1 - \frac{C'(\lambda)\lambda}{C(\lambda)})}
\end{aligned}$$

Note that $\dot{C} = \frac{dC}{dt}$ and $C' = \frac{dC}{d\lambda}$. With assumption that λ is in the neighborhood of 90%,

$$1 - \frac{C'(\lambda)\lambda}{C(\lambda)} \neq 0$$

$$\therefore \begin{cases} \frac{\dot{\omega}}{\omega} & = \Phi(\lambda) - \alpha \\ \frac{\dot{\lambda}}{\lambda} & = \frac{C(1-\omega) - \nu^* \cdot (\alpha+\beta+\delta)}{\nu^*(1 - \frac{C'(\lambda)\lambda}{C(\lambda)})} \end{cases} \qquad (3)$$

Based on the data file we collected from OECD countries, we can calculate the following parameters. We found $\alpha \approx 1.5\%$ $\quad \beta \approx 1\%$ $\quad \delta \approx 6.5\%$ $\quad \omega^* \approx 60\%$ $\quad \lambda \approx 93\%$ and $C'(0.93) \approx 2.5$.With this result, we found that

$$1 - \frac{C'(\lambda)\lambda}{C(\lambda)} < 0$$

The model, therefore, is unstable.

2 Economics Capacity and Capacity Utilization Theory by Shaikh

Shaikh define output as following:

$$Y = \frac{Y}{Y^*} \cdot \frac{Y^*}{K} \cdot K \qquad (4)$$

where Y^* is economics capacity, which is a desired level of output from capital stock or the benchmark level of output (imaginary). He then define:

$$\begin{aligned} u &= \frac{Y}{Y^*} \qquad \text{Capacity Utilization Rate} \\ v &= \frac{K}{Y^*} \qquad \text{Capital-Capacity Ratio} \end{aligned} \qquad (5)$$

Therefore, the ouput Y can be rewritten as:

$$Y = \frac{K \cdot u}{v} \tag{6}$$

$$\log Y(t) = \log K(t) - \log v(t) + \log u(t) \tag{7}$$

We assume that the output fluctuates around capacity over the longrun, so that the actual rate of capacity utilization $u(t)$ fluctuates around some desired or normal rate of capacity utilization (u^*).

$$\log u(t) = e_u \tag{8}$$

Shaikh's second behavioral assumption consists of a general specification of technical change in which capacital-capacity ratio($v(t)$) changes over time, partly in response to autonomous technical change (coefficient b_1) and partly in response to embodied technical change which itself depends on the rate of capital accumulation (coefficent b_2). Let:
g_v: growth rate of the capital-capacity ratio
g_k: growth rate of the capital stock.
Then

$$\begin{aligned} g_v &= b_1 + b_2 \cdot g_k \\ \Rightarrow \log v(t) &= b_0 + b_1 \cdot t + b_2 \cdot \log K(t) + e_v \\ \therefore \log Y(t) &= a_0 + a_1(t) + a_2(t) \cdot \log K(t) + e(t) \end{aligned} \tag{9}$$

where $a_0 = -b_0 \qquad a_1 = -b_1 \qquad a_2 = 1 - b_2$. From equation(9) the logarithm value of actual output Y is the economics capacity Y^*. With an estimation of capacity in hand, we can then derive the rate of capacity utilization $u(t) = \frac{Y}{Y^*}$ and the capital-capacity ratio $(v(t) = \frac{K}{Y^*}$. According to data we collect from OECD countries, we assume that u is an increasing funciton of λ. Since the range of λ in the real world is so small and at the neigborhood of 93 %, we assume that

$$u(\lambda) = Q + R \cdot \lambda \tag{10}$$

We derive the equation as following:

$$
\begin{aligned}
\text{Note that} \qquad Y &= \frac{K \cdot u}{v} \\
&= L \cdot a \\
&= N \cdot \lambda \cdot a. \\
\dot{K} &= Y \cdot (1-\omega) - \delta \cdot K \\
\text{Then} \qquad \frac{\dot{Y}}{Y} &= \frac{\dot{K}}{K} + \frac{\dot{u}}{u} - \frac{\dot{v}}{v} \\
\Rightarrow \frac{\dot{\lambda}}{\lambda} + \frac{\dot{a}}{a} + \frac{\dot{N}}{N} + &= \frac{Y \cdot (1-\omega) - \delta K}{K} + \frac{u'(\lambda)}{u(\lambda)}\dot{\lambda} - \left(b_1 + b_2 \cdot \left(\frac{u \cdot (1-\omega)}{v} - \delta\right)\right) \\
\Rightarrow \frac{\dot{\lambda}}{\lambda} + \alpha + \beta &= \frac{u \cdot (1-\omega)}{v} - \delta + \frac{u'(\lambda)}{u(\lambda)}\dot{\lambda} - \left(b_1 + b_2 \cdot \left(\frac{u \cdot (1-\omega)}{v} - \delta\right)\right) \\
\Rightarrow \frac{\dot{\lambda}}{\lambda} - \frac{u'(\lambda)}{u(\lambda)}\dot{\lambda} &= -b_1 + (1-b_2) \cdot \left(\frac{u \cdot (1-\omega)}{v} - \delta\right) - \alpha - \beta \\
\Rightarrow \frac{\dot{\lambda}}{\lambda} &= \frac{-b_1 + (1-b_2) \cdot \left(\frac{u \cdot (1-\omega)}{v} - \delta\right) - \alpha - \beta}{(1 - \frac{u'(\lambda)\lambda}{u(\lambda)})}
\end{aligned}
\tag{11}
$$

$$
\therefore \begin{cases}
\frac{\dot{\omega}}{\omega} &= \Phi(\lambda) - \alpha \\
\frac{\dot{\lambda}}{\lambda} &= \frac{-b_1 + (1-b_2) \cdot \left(\frac{u \cdot (1-\omega)}{v} - \delta\right) - \alpha - r\beta}{(1 - \frac{u'(\lambda)\lambda}{u(\lambda)})} \\
\frac{\dot{v}}{v} &= b_1 + b_2 \cdot \left(\frac{u \cdot (1-\omega)}{v} - \delta\right)
\end{cases}
\tag{12}
$$

Similar in section I, we calculate the following parameters with the data we have from OECD countries. We found $\alpha \approx 1.5\%$ $\beta \approx 1.5\%$ $\delta \approx 10\%$ $b_1 \approx -0.025$ $b_2 \approx 0.809$ $\lambda \approx 93.5\%$ $u(0.935) \approx 1$ $u'(0.935) \approx 2.4$ $u(0) \approx 1$ and $v(0) \approx 2.5$. Based on this result, we found that

$$1 - \frac{u'(\lambda)\lambda}{u(\lambda)} < 0$$

The model, therefore, is unstable.

3 References

1. M.Shaikh,A. and J.Moudud,(2004),"Measuring Capacity Utilization in OECD countries:A Cointegration Method".Working Pape No.415 at The Levy Economics Institute of Bard College.

2. Keen,Steve(1997),"Economic Growth and Financial Instability",A thesis submitted for the degree of Doctor of Philosophy at The University of New South Wales:79-85