ABSTRACTS of the Conference “Legacy of Vladimir Arnold”

The Fields Institute, November 24-28, 2014

D. Bar-Natan

Finite type invariants of doodles

Abstract: I will describe my former student's Jonathan Zung work on finite type invariants of "doodles", plane curves modulo the second Reidemeister move but not modulo the third. We use a definition of "finite type" different from Arnold's and more along the lines of Goussarov's "Interdependent Modifications", and come to a conjectural combinatorial description of the set of all such invariants. We then describe how to construct many such invariants (though perhaps not all) using a certain class of 2-dimensional "configuration space integrals".

and

Dessert: Hilbert's 13th Problem, in full color (special expository talk)

Abstract: To end a week of deep thinking with a nice colorful light dessert, we will present the Kolmogorov-Arnol'd solution of Hilbert's 13th problem with lots of computer-generated rainbow-painted 3D pictures.

In short, Hilbert asked if a certain specific function of three variables can be written as a multiple (yet finite) composition of continuous functions of just two variables. Kolmogorov and Arnol'd showed him silly (ok, it took about 60 years, so it was a bit tricky) by showing that ANY continuous function f of any finite number of variables is a finite composition of continuous functions of a single variable and several instances of the binary function "+" (addition). For $f(x,y) = xy$, this may be $xy = \exp(\log x + \log y)$. For $f(x,y,z) = x^y/z$, this may be $\exp(\exp(\log y + \log\log x) + (-\log z))$. What might it be for (say) the real part of the Riemann zeta function?

The only original material in this talk will be the pictures; the math was known since around 1957.

O. Bogoyavlenskij

A family of solutions to the Arnold's problem on the non-degenerate integrability

Abstract: Exact solutions to the steady Euler equations are constructed. It is proved that the corresponding dynamics of ideal fluid is integrable, preserves invariant tori T^2 and is non-degenerate. The streamlines on almost all tori T^2 are quasi-periodic infinite curves which are dense on the tori and there is a dense countable subset of tori T^2 on which all streamlines are closed curves which generically are non-trivial knots in the space $\mathbf{R}^3$. Exact solutions to the Navier-Stokes equations with a time-dependent piece-wise continuous viscosity $\nu(t)$ are derived. For the exact solutions the total path length of any streamline for $t \geq 0,\ t < \infty$ is finite and therefore the corresponding fluid flows are not turbulent for any bounded Reynolds number R. The finiteness of the total path length is proven also for some exact solutions with time-dependent Reynolds number $R(t)$ in the case of unbounded growth $R(t) \to \infty$ when $t \to \infty$. The dynamics of fluid for these solutions also is not turbulent in spite of the Reynolds number $R(t)$ tends to infinity.

S. Chmutov

Partial duality of hypermaps

Abstract: Hypermaps differ from graphs on surfaces in a way that their edges are allowed to connect more then two vertices. Orientable hypermaps arise naturally in the Grothendieck's dessins d'enfants theory.

We introduce a collection of new operations on hypermaps, partial duality, which include the classical Euler-Poincaré dualities as particular cases. These operations generalize the partial duality for maps, or ribbon graphs, recently discovered in a connection with the knot theory.

Combinatorially hypermaps may be described in one of three ways: as three involutions on the set of flags (τ-model), or as three permutations of the set of half-edges (σ-model in orientable case), or as edge 3-colored graphs. We express partial duality in each of these models. This is a joint work with Fabien Vignes-Tourneret (Lyon). The details are in `arXiv:1409.0632 [math.CO]`.

W. Craig

Vortex filament dynamics

Abstract: This talk is on a problem in mathematical hydrodynamics, describing phenomena observed in solutions of the Euler equations of hydrodynamics in a setting with a strong analogy to Hamiltonian dynamical systems. The problem addresses a system of model equations for the dynamics of near-parallel vortex filaments in a three dimensional fluid. These equations can be formulated as a Hamiltonian system of partial differential equations, and the talk will describe some aspects of a phase space analysis of solutions, including a theory of periodic and quasi-periodic orbits via a version of KAM theory, and a topological principle to count multiplicity of solutions. This is ongoing joint work with C. Garcia and C.-R. Yang (McMaster and The Fields Institute).

G. Domokos

The Gömböc and the evolution of pebble shapes

Abstract: In 1995, V.I. Arnold conjectured that convex, homogeneous solids with just two static balance points may exist. Ten years later, based on a constructive proof, the first such object (dubbed "Gömböc") was built. The first numbered Gömböc (Gömböc 001) was given to V.I. Arnold on the occasion of his 70th birthday in Moscow. Here Arnold proposed that the Gömböc may play a role in explaining the geometric evolution of pebbles.

It appears that, once again, he may have been right. The best known mathematical models for the abrasion of sedimentary particles are curvature-driven flows: a special class of nonlinear partial differential equations defining the evolution of a surface Σ by the speed v in the direction of its surface normal, and v is given as a function of the principal curvatures κ, λ of Σ:

$$v = v(\kappa, \lambda).$$

While locally defined, curvature-driven flows have startling global properties, e.g., they can shrink curves and surfaces to round points. These features made these flows

powerful tools to prove topological theorems which ultimately led, via their generalizations by Hamilton to Perelmans's celebrated proof of the Poincaré conjecture.

As a by-product of these great efforts, in 1987 Grayson proved that if Σ is given as a distance function from a fixed reference O then the number $N(t)$ of spatial critical points (extrema of the distance) is decreasing monotonically under the planar $v = \kappa$ flow, also called the curve shortening flow.

We will show that there is mounting evidence that similar, though weaker (generic, stochastic) statements are true for static balance points on 3D solids evolving under the above displayed v: based on some results on curvature-driven flows we propose a Markov process governing the evolution of the number of static balance points. Our Markov process is similar in spirit to the stochastic models adopted in digital image processing to track the number of critical points.

Our model predicts that the expected value of the number of static balance points on abrading solids will decrease monotonically. Laboratory and field data show a remarkable match with the proposed Markov process. We will also discuss why Gömböc shapes, with minimal number of balance points, are almost never found in Nature. Rather, as physicist Sir Michael Berry expressed it, "they exist in Nature only as a dream".

V. Dragovic

Algebro-geometric approach to the Schlesinger equations and the Poncelet polygons

Abstract: In 1995 Hitchin constructed explicit algebraic solutions to the Painlevé VI (1/8,-1/8,1/8,3/8) equation starting with any Poncelet polygon inscribed in a conic and circumscribed about another conic. We will show that Hitchin's construction can be generalized and embedded in the Okamoto transformation between Picard's solution and the general solution of the Painlevé VI (1/8,-1/8,1/8,3/8) equation. Moreover, we will show that this Okamoto transformation can be presented in an invariant, geometric, way, in terms of an Abelian differential of the third kind on the associated elliptic curve. The last observation allows us to obtain solutions to the corresponding 2×2 Schlesinger system with four poles in terms of this differential as well. The solution of the Schlesinger system admits a natural generalization to the case with $2g + 2$ poles and corresponding Riemann surfaces of genus g. These higher genera solutions, specialized to the case of rational parameters, are related to higher-dimensional Poncelet polygons associated to g confocal quadrics in $g + 1$ dimensional space, closing the loop with the initial Hitchin's remarkable observation. This is a joint work with V. Shramchenko.

Ya. Eliashberg

Construction and classification of contact structures

Abstract: I will discuss a recent progress in construction an classification of overtwisted contact structures in all dimensions. This is a joint work with M.S. Borman and E. Murphy.

P. Etingof

Poisson homology, D-modules on Poisson varieties, and complex singularities

Abstract: Let X be a complex affine Poisson variety with finitely many symplectic leaves, and $O(X)$ be the algebra of polynomial functions on X. I will explain why the zeroth Poisson homology $O(X)/\{O(X), O(X)\}$ is finite dimensional. The proof is based

on attaching to X a right D-module $M(X)$ on X which is the quotient of the canonical D-module $D(X)$ on X by the action of Hamiltonian vector fields, and showing that it is holonomic, and that the zeroth Poisson homology of X is just the underived direct image of $M(X)$ to the point. This motivates considering the full direct image of $M(X)$ in the more general case when X is not necessarily affine. This direct image is called the Poisson-de Rham homology of X. If X is symplectic, then $M(X)$ is the canonical sheaf, and thus the Poisson-de Rham homology coincides with the usual de Rham cohomology of X, but in general the Poisson-de Rham homology is hard to compute. However, if X has a symplectic resolution of singularities, we conjectured with T. Schedler that the Poisson-de Rham homology of X is isomorphic to the de Rham cohomology of the resolution. I will explain the motivation for this conjecture, and describe a few cases when it is proved (symmetric powers of simple surface singularities, Springer fibers). I will also describe some other cases (such as surfaces in $\mathbf{C}^3$ with isolated singularities) when there is no symplectic resolution but there is a smooth symplectic deformation, and the Poisson-de Rham homology equals the cohomology of the deformation. This is joint work with T. Schedler.

E. Ferrand

Remarks and questions about apparent contours, enveloppes and generating families

D. Fuchs

Cohomology of Lie algebra of Hamiltonian vector fields: experimental data, conjectures, and theorems

A. Gabrielov

Classification of spherical quadrilaterals

Abstract: A spherical quadrilateral (membrane) is a bordered surface homeomorphic to a closed disc, with four distinguished boundary points called corners, equipped with a Riemannian metric of constant curvature 1, except at the corners, and such that the boundary arcs between the corners are geodesic. We discuss the problem of classification of these quadrilaterals and perform classification up to isometry in the case that at most three angles at the corners are not multiples of π. This is a very old problem, related to the properties of solutions of the Heun's equation (a second order linear differential equation with four regular singular points). The corresponding problem for the spherical triangles, related to the properties of solutions of the hypergeometric equation, has been solved by Klein, with some gaps in Klein's classification filled in by Eremenko in 2004. Classification of quadrilaterals with arbitrary corners remains an open problem.

V. Ginzburg

The motion of a charge in a magnetic field

Abstract: I am going to talk on applications of symplectic topology to Hamiltonian dynamics and, more specifically, about the motion of a charge in a magnetic field. The area owes its existence to Arnold, it has been quite active since his original work, and there are some pretty recent developments.

A. Givental

Oscillating integrals in the mirror K-theory

Abstract: By way of examples, I will first recall how complex oscillating intergrals arise in quantum cohomology theory, and then discuss a novel kind of oscillating integrals arising in connection with quantum K-theory, q-difference equations, and q-hypergeometric functions.

V. Goryunov

Local invariants of maps between 3-manifolds

Abstract: We classify order 1 invariants of maps between 3-manifolds whose increments in generic homotopies are defined entirely by diffeomorphism types of local bifurcations. We show that in the oriented case the space of integer invariants has rank 7 for any source and target, and give a geometric interpretation of its basis. The mod2 setting, with ?3 as the target, adds another 4 linearly independent invariants. One of them combines the self-linking of the cuspidal edge of the critical value set **C** with the number of connected components of the edge, and further two are similar combinations for the links constructed from the edges and self-intersection locus of **C**.

Time permitting, we will also analyze general non-oriented settings and obtain either exact descriptions or rank estimates for the spaces of integer- and mod2-valued invariants.

S. Gusein-Zade

Some duality properties of invertible polynomials

Abstract: Arnold's strange duality between exceptional unimodal hypersurface singularities seems to be the first observation of a mirror symmetry effect. A generalization of it is the Berglund-Hübsch-Henningson duality between so-called invertible polynomials. They appeared as superpotentials in mirror-symmetric Landau-Ginzburg models. In the orbifold (Berglund–Henningson) setting this duality treats a pair consisting of an invertible polynomial and a (finite abelian) group of its symmetries together with a dual pair. We shall describe some dualities related with the actions of the symmetry groups on the corresponding Milnor fibres.

The talk is based on joint works with W. Ebeling.

Yu. Ilyashenko

Rotation numbers and moduli of elliptic curves

Abstract: A new fractal set of "complexified Arnold's tongues" will be described. It occurs in the following way. Consider an analytic diffeomorphism f of a unit circle S^1 into itself. Let $f_\lambda = \lambda f$, and $|\lambda| \leq 1$. If $|\lambda| = 1$, then f_λ is still an analytic diffeomorphism of a circle into itself; let $\rho(\lambda)$ be its rotation number. If $|\lambda| < 1$, then an elliptic curve occurs as a factor space of an action of f_λ. Let $\mu(\lambda)$ be the "multiplicative modulus" of this elliptic curve. A "moduli map" of unit discs $\lambda \to \mu(\lambda)$ occurs. The problem is to describe its limit values.

It appears that the boundary values of the moduli map form a fractal set: a union of S^1 and a countable number of "bubbles' adjacent to all the roots of unity from inside S^1. Relations of these limit values and rotation numbers $\rho(\lambda)$ will be described.

These results are motivated by problems stated by Arnold and Yoccoz, and are due to Risler, Moldavskis, Buff, Goncharuk and the speaker. Some open problems will be stated.

L. Jeffrey
Kirwan surjectivity for hyperkähler manifolds
Abstract: Let M be a hyperkähler manifold equipped with an action of a compact Lie group K which is Hamiltonian with respect to all three symplectic forms $(\omega_1, \omega_2, \omega_3)$ of M. Let $X = M//K$ be the Kähler quotient (with respect to ω_1) and let $M = M////K$ be the hyperkähler quotient. Define $\mu = \mu_{\mathbf{R}} \oplus \mu_{\mathbf{C}}$. If 0 is a regular value of μ, the Kirwan map is defined by $\kappa : H^*_K(M) \to H^*_K(\mu^{-1}(0)) \cong H^*(\mu^{-1}(0)/K)$. We describe hypotheses under which κ is surjective, and outline the proof of surjectivity. The method gives surjectivity except possibly in middle degree. (Joint work with Jonathan Fisher, Young-Hoon Kiem, Frances Kirwan and Jon Woolf).

V. Kaloshin
On Arnold diffusion for convex Hamiltonians
Abstract: In 1964 Arnold constructed an example of instability for nearly integrable Hamiltonian systems and conjectures that this is a generic phenomenon. During the talk I will describe a recent progress proving this conjecture for convex Hamiltonians in any number of degrees of freedom. This is joint work with Ke Zhang.

Y. Karshon
TBA

M. Kazarian
TBA

K. Khanin
From linearization to rigidity theory
Abstract: In his seminal 1961 paper Arnold founded an area of smooth linearization for circle diffeomorphisms. In this talk we shall present the modern state of the field. We shall also discuss its connection with renormalization and rigidity theory.

S. Lando
On graph invariants related to finite order knot invariants
Abstract: Finite order, or Vassiliev, knot invariants can be understood as functions on chord diagrams satisfying certain relations (weight systems). To a chord diagram, a simple graph can be associated, which is the intersection graph of the diagram. This construction allows one to obtain knot invariants from certain graph invariants, and the natural question arises what are graph invariants that produce knot invariants. In particular, the chord diagram structure allows one to assign signs to circuits of even length in this graph in a natural way. The difference between the number of positive and negative circuits of given length $2k$ is a graph invariant. This invariant is closely related to the weight system associated to the Lie algebra sl_2. It happens that this invariant admits a natural extension to arbitrary graphs, including those that are not intersection graphs of chord diagrams.

This fact leads to a number of questions concerning the existence of more graph invariants related to sl_2 and other Lie algebras. The talk is based on a joint paper with E. Kulakova, T. Mukhutdinova and G. Rybnikov (2014).

G. Misiolek

The Morse-Littauer theorem in hydrodynamics

Abstract: The right-invariant L^2 metric on the group of volume-preserving diffeomorphisms has a well de fined exponential map whose properties are not yet fully understood. Even in finite dimensional geometry singularities of Riemannian exponential maps are of a special kind. In 1932 Morse and Littauer showed that singularity of $d\exp_p(v)$ necessarily implies non-injectivity of $\exp_p$ near v. I will describe an analogue of that theorem in 2D hydrodynamics. The proof relies on the fact that the exponential map is a Fredholm map of index zero.

K. Moffatt

Relaxation under topological constraints and the breaking of these constraints

Abstract: Dynamical systems have a natural tendency to relax through dissipative processes to a minimum-energy state, subject to relevant topological constraints. An example is provided by the relaxation of a magnetic field in a perfectly conducting but viscous fluid, subject to the constraint that the magnetic field lines are frozen in the fluid, a situation first conceived by Arnol'd (1974). One may infer the existence of magnetostatic equilibria (and analogous steady Euler flows) of arbitrary field-line topology. In general, discontinuities (current sheets) appear during this relaxation process, and this is where reconnections of field-lines (with associated change of topology) can and do occur.

An analogy is provided by the dynamics of a soap-film bounded by a flexible wire (or wires) which can be continuously and slowly deformed. At each instant the soap-film relaxes in quasi-static manner to a minimum-area (i.e. minimum-energy) state compatible with the boundary configuration. This can however pass through a critical configuration at which a topological jump is inevitable. We have studied an interesting example of this behaviour: the jump of a one-sided (Möbius strip) soap-film to a two-sided film as the boundary is unfolded and untwisted from the double cover of a circle. The nature of this jump will be demonstrated and explained.

M. Polyak

From 3-manifolds to planar graphs and cycle-rooted trees

Abstract: We describe an encoding of 3-manifolds by weighted planar graphs. Knots also naturally fit in the picture. This encoding turns out to be intimately related to a number of interesting topics: graph Laplacians, electric networks, etc. Furthermore, a number of interesting knot and 3-manifold invariants can be obtained by a weighted counting of subgraphs. We describe the simplest invariants arising from a discrete version of a perturbative Chern-Simons theory.

I. Scherbak

Linear ODEs from an algebraic point of view

Abstract: The talk is devoted to the algebraic approach proposed by L. Gatto and myself to treat the generic linear ODE

$$u^{(r)}(t) - e_1 u^{(r-1)}(t) + ... + (-1)^r e_r u(t) = 0,$$

where $e_1, ..., e_r$ are indeterminates. We solve it in the ring $B_r[[t]]$ of formal power series with coefficients in the polynomial ring $B_r = \mathbf{Q}[e_1, ..., e_r]$.

The role Wronskians play in Schubert calculus on Grassmannians was the initial motivation of our study. A fundamental system of solutions to our ODE defines the e_j's in terms of generalized Wronskians of the system.

The generalized Wronskians are labeled by partitions and are given by acting on the usual Wronskian via Schur polynomials. Thus, the free module generated by the generalized Wronskians is naturally isomorphic to the singular homology module of the Grassmannian of r-planes in a vector space. In this setting, Pieri?s formula of Schubert calculus is nothing but the Leibniz rule for derivation of generalized Wronskians.

Moreover, the infinite-order generic linear ODE (having countably infinitely many indeterminate coefficients $e_1, e_2, ...$) is related to representations of the Heisenberg Oscillator Algebra. Namely, the bosonic space B_∞ can be realized as the $\mathbf{Q}$-vector space generated by the solutions, and the boson-fermion corre- spondence for the zero charge fermionic space is a consequence of the universal Cauchy formula expressing each solution to the generic linear ODE as a linear combination of the elements of the universal basis of solutions.

S. Shandarin

Zeldovich-Arnold's theory of the Cosmic Web

Abstract: Astronomical observation of the last three decades have shown that galaxies are not distributed in the universe statistically uniformly. Instead they form an intricate structure ? dubbed as the Cosmic web. The Cosmic web consists of four generic elements: compact clusters of galaxies connected by the filaments which are crossings of the walls and voids - the regions between the walls. The density of galaxies is the highest in the clusters and the lowest in the walls. In 1970 Ya.B. Zeldovich suggested a simple mathematical model - the Zeldovich approximation - that predicted the main features of the Cosmic web. As shown by V.I. Arnold in 1981 the Zeldovich approximation applied to cold continuous collisions medium - an excellent approximation describing dark matter in the universe - results in emergence of a set of caustics at the nonlinear stage. This set of caustics can be associated with the elements of the Cosmic web. I will present a brief review of the current state of the theory of the Cosmic web based on Zeldovich-Arnold?s model.

M. Shapiro and A. Vainshtein (double talk)

Cluster algebras and integrable systems

Abstract: We are going to discuss the Arnold problem on the number of connected components for complete real flags, after Berenstein, Fomin, Zelevinsky, Postnikov's networks, cluster algebras, and integrable systems, with the pentagram map as an example.

A. Shnirelman

Shooting problem for the 2D ideal incompressible fluid

Abstract: Let M be a smooth compact Riemannian 2-d manifold, and $s > 3/2$. Let V^s denote the space of divergence-free vector fields of Sobolev class H^s; let D^s be the space of volume-preserving diffeomorphisms of M of class H^s. This is a smooth infinite-dimensional weakly Riemannian manifold (with respect to the L^2-metric). V.I.Arnold stated that the fluid flows in M can be regarded as geodesics on D^s; by the Wolibner-Kato-Yudovich theorem, g_t exists for all $t \in \mathbf{R}$. If $g_0 = Id$, the identity map, and $v_0 = \dot{g}_0 \in V^s$, then we define the geodesic exponential map $\mathrm{Exp} : v_0 \mapsto g_1$, $\mathrm{Exp} : V^s \to D^s$.

Theorem: $\mathrm{Exp}(V^s) = D^s$

This statement looks superficially like an infinite-dimensional version of the Hopf-Rinow Theorem. However, it has little to do with it. In fact, the Hopf-Rinow Theorem establishes the existence of the *minimal* geodesic connecting two points on a (geodesically complete) Riemannian manifold. But a minimal geodesic on D^s connecting two fluid configurations can not exist at all while some (non-minimal) geodesic certainly exists. (This is associated with the existence of conjugate points in D^s found by G. Misiolek.) The proof of this theorem is based on combination of ideas which can be called "micro-global analysis".

V. Timorin

On maps taking lines to plane curves

Abstract: A planarization is a mapping f of an open subset U of the real projective plane into the real projective n-space, such that $f(L \cap U)$ is a subset of a hyperplane, for every line L. Studying planarizations is closely related to studying maps taking lines to curves of certain linear systems; a classical result of this type is the Moebius–von Staudt theorem about maps taking lines to lines, sometimes called the Fundamental Theorem of Projective Geometry. We assume that the planarizations are sufficiently smooth, i.e., sufficiently many times differentiable. We give a complete description of all planarizations in case $n = 3$ up to the following equivalence relation: two planarizations are equivalent if they coincide on a nonempty open set, after a projective transformation of the source space and a projective transformation of the target space. Apart from trivial cases, there are 16 equivalence classes, among which 6 classes of cubic rational maps (all remaining nontrivial classes are represented by quadratic rational maps).

A. Vershik

Dynamics of metrics and scaling entropy of the group action

O. Viro

Head to tail compositions

Abstract: Any isometry of the Euclidean space can be presented as a composition of two reflections in subspaces. In low dimensions this provides a convenient graphical encoding of isometries similar to (and, in a sense, generalizing) the encoding of translations by arrows and composition of translations to the head to tail addition of vectors. Non-uniqueness of representation for rotations and translations as compositions by two reflections provides a simple complete system of relations among the reflections. Generalizations to other classical geometries also will be discussed.

V. Vladimirov

Arnold stability of time-oscillating flows

Abstract: A new theory of oscillating in time flows is developed. The flows are inviscid and incompressible, the flow oscillations are introduced by the oscillating boundary conditions. We use the two-timing method, where the small parameter is the ratio of the 'slow' and 'fast' time-scales, where the 'fast' time-scale is prescribed by the boundary oscillations. Our attention is focused on the derivation of the averaged equations of motion and on the obtaining of the Arnold stability criteria for the averaged flows. The results are:

The analysis of distinguished limits for the Euler's equations shows the existence of only two asymptotic models for the averaged flows: (VD) – the standard vortex dynamics, and (CLD) – the generalized Craik-Leibovich dynamics. The VD states that the averaged vorticity is 'frozen' into the averaged velocity, while the CLD shows that the averaged vorticity is 'frozen' into the averaged velocity+drift velocity. In the case we consider, the drift velocity has the same order of magnitude as the averaged velocity. Our derivation of the CLD is much simpler technically than all the previous derivations. The formulation of the problem in its natural generality shows that the area of applicability of the CLD is much broader than it has been targeted before. In particular, our generic flow domain is three-dimensional and arbitrary; the oscillations are time-periodic, but their spatial structure is arbitrary. The slow time-scale is uniquely linked to the magnitude of the prescribed velocity field at the boundary. We also have derived the averaged boundary conditions related to the oscillating boundaries.

The CLD operates with the generalized 'isovorticity conditions' and and leads to the energy-type integral for the averaged flows, which allows us to consider the 'Arnold-type' results, such as the energy-type variational principle, the first and second variation of the energy, and several (nonlinear and/or linear) stability criteria for averaged flows.

In both cases (VD) and (CLD) the next-order approximation (after the main ones) represent the linearized equations with an additional 'force', which produces additional instabilities.

The relations of the obtained results to the Langmuir circulations and to the MHD dynamo are discussed.

A. Zorich

Lyapunov exponents of the Hodge bundle and diffusion in periodic billiards

Abstract: Asymptotic behavior of leaves of a measured foliation on a Riemann surface is governed by the mean monodromy of the Hodge bundle along the associated trajectory of the Teichmüller geodesic flow in the moduli space. As a consequence, recent progress in the study of the Teichmüller flow (inspired by the fundamental works of A. Eskin, M. Mirzakhani, and A. Mohammadi) and in the study of the Lyapunov exponents of the Hodge bundle along this flow leads to new results on measured foliations on surfaces.

Following ideas of V. Delecroix, P. Hubert, and S. Lelièvre I will show how to apply this technique to description of the diffusion of billiard trajectories in the plane with periodic polygonal obstacles. The results presented in the talk are obtained in collaboration with J. Athreya, V. Delecroix, A. Eskin, and M. Kontsevich.