# LECTURE SCHEDULE

**International Conference** 

# GROUPS, RINGS AND GROUP RINGS

July 28–August 1, 2014

Brock University , St. Catharines , Canada

Organizers: Yuanlin Li (Brock University), Allen Herman (University of Regina), Eric Jespers (Vrije Universiteit Brussel), Wolfgang Kimmerle (Universität Stuttgart)

### LECTURE ROOMS (Academic South=AS)

Room A = AS 216 (Invited talks) Room B = AS 217

Sunday: July 27 - Arrival day

Saturday: August 2 - Departure day

### **Invited Speakers**

- Eli Aljadeff (Technion University, Israel )
- Gurmeet K. Bakshi (Panjab University)
- Jason Bell (University of Waterloo, Canada)
- Yuri Bahturin (Memorial University, Canada)
- Gabriele Nebe (RWTH Aachen, Germany)
- Jan Okninski (Warsaw University, Poland)
- César Polcino Milies (University of Sao Paulo, Brazil)
- David Riley (Western University, Canada)
- Ángel del Río (University of Murcia, Spain)
- Benjamin Steinberg (City College of New York, USA)
- Sergio Lopez-Permouth (University of Ohio, USA)

## Monday: July 28

8:45-9:30 an	n On site registration		
9:30-10:00 an	n Coffee		
10:00-10:05 an	n Opening of conference		
10:10-11:10 an	n <i>Eli Aljadeff</i>		
Room A	On a G-graded version of Jordans theorem		
	Room A	Room B	
11:15-11:35 an	A Presentation for a Group Acting Discontinuously on a Direct Product of Copies of	a Group Rings	
11:40-12:00 an	<ul> <li>Hyperbolic Spaces</li> <li>Maldemar Holubowski</li> <li>Matrix representations of a free group</li> </ul>	Yakov Karasik e The Co-dmension Sequence of Finite Dimensional G Simple Graded Algebras	
12:00 pm	Lunch		
2:00-3:00 pm Room A 3:00-3:30 pm	Gabriele Nebe Computing unit groups of orders Coffee break		
	Room A	Room B	
3:30-3:50 pm	Kenza Guenda Ring's Application in Informa- tion Theory	Hongdi Huang On (5, 18)groups	
3:55-4:15 pm	Florian Eisele Units of integral group rings of finite groups up to commensu- rabilty	Firdousi Begam On Submodule Transitivity of QTAG-module	
4:20-4:40 pm	<i>Ofir David</i> <b>Regular G-gradings on algebras</b>	Liang Shen A note on the Faith-Menal con- jecture	
4:45-5:05 pm	<i>Marcin Mazur</i> Genrators of maximal order	Jung Wook Lim S-Noetherian properties in a special pullback	

0.00 10.00 4111	29 Gurmeet K. Bakshi A complete irredundant set of strong Shoda pairs
10:00-11:00 am Room A	Yuri Bahturin Relative growth of subsets in free groups and algebras
11:00-11:30  am	Coffee break
11:30-12:15 am Room A	Eric Jespers Krull orders in nilpotent groups
12:15 pm	Lunch
2:00-3:00 pm Room A	David Riley Behaviour of the Frobenius map in a noncommutative world
3:00-3:30 pm	Coffee break

	Room A	Room B
3:30-3:50 pm	Geoffrey Janssens $ZS_n$ modules and polynomial identities with integer coeffi- cients	Allan Berele What are double centralized theorems good for?
3:55-4:15 pm	Manizheh Nafari AS-Regular Graded Skew Clif- ford Algebras that are Twists of AS-Regular Graded Clifford Algebras	Jeffrey Bergen Subrings of invariants of do- mains with finite Gelfand- Kirillov dimension
4:20-4:40 pm	Basil Nanayakkara Existence of terminal resolu- tions of geometric Brauer pairs in arbitrary dimension	Jiangling Chen Some *-clean group rings
4:45-5:05 pm	Chris Plyley On the Relationship of Actions and Gradings	<i>Tsunekazu Nishinaka</i> Primitivity of group algebras o non-noetherian groups
5:10-5:30 pm	Stefan Catoiu Pythagorean Counting in Rank One Restricted Enveloping Al- gebras	Mayada Shahada Multiplicatively Collapsing and Rewritable Algebras
6:00 pm <b>(</b> Pond Inlet	Cash bar	
6:30 pm <b>(</b> Pond Inlet	Conference dinner	

Wednesday: July 30

Day trip to Niagara Falls, by bus.

Morning: visit to the Falls (boat trip, pay yourself).

Lunch from 12:20 - 1:20 pm.

Visit to local winery and other visits.

## Thursday: July 31

9:00-10:00 am Room A		ngs	
10:00-11:00 am Room A	5	in group rings	
11:00-11:30 am	n Coffee break		
11:30-12:30 Room A	5 1	nsist solely of units	
12:30 pm	n Lunch		
	2:00-3:00 pm Jan Okninski Room A <b>Prime ideals in algebras determined by submonoids</b> of nilpotent groups		
3:00-3:30 pm	n Coffee break		
	Room A	Room B	
3:30-3:50 pm	Inneke Van Gelder Idempotents in group algebras over number fields	Alvaro P. Raposo Clean rings and group algebras. Part I	
3:55-4:15 pm	Jairo Z. Goncalves Free groups in normal sub- groups of the multiplicative group of a division ring	Paula Murgel Veloso Clean rings and group algebras. Part II	
4:20-4:40 pm Room A	Maya Van Campenhout Finitely generated algebras defined by homogeneous quadratic monomial relations and their underlying monoids		
4:45-5:05 pm Room A	Allen Herman How to identify division algebras in the Wedderburn Decomposition of QG using GAP		

#### Friday: August 1

9:00-10:00  am	Benjamin Steinberg
Room A	Etale groupoid algebras
	Cesar Polcino Milies Essential idempotents in group algebras and minimal cyclic codes
11:00-11:30 am	Coffee break

- Room A Room B 11:30-11:50 am Hamid Usefi Gurmail Singh Classification of finite dimen-Lagrange type theorems for the sional p-nilpotent restricted Lie torsion units of integral adjaalgebras cency rings of finite association schemes 11:55-12:15 pm Daniel Smertnig Mohamed A. Salim Factorization theory in maxi-On the unit group of a commumal orders in central simple altative group ring gebras 12:15 pm Lunch 2:00-2:20 pm Sugandha Maheshwary Room A Wedderburn Decomposition of Rational group algebras - A computational approach 2:25-2:45 pm Andreas Bächle Room A On the prime graph question for 4-primary groups I 2:50-3:10 pm Leo Margolis Room A On the prime graph question for 4-primary groups II 3:15-4:00 pm Wolfgang Kimmerle Room A Sylow properties of a finite group determined by its (group) rings
  - 4:05 pm Closing Ceremony

### ABSTRACTS

### Monday

Eli Aljadeff, Technion-Israel Institute of Technology, Haifa,

#### *Title:* On a *G*-graded version of Jordan's theorem

A well known theorem of Camille Jordan (1878) says that if G is a finite group which may be embedded in  $GL_n(\mathbb{C})$  then it is "almost abelian" in the sense that it contains an abelian subgroup H whose index [G:H] is bounded by a function of n.

The main object of this lecture is to present an analogous result for G-graded algebras where G is arbitrary (i.e. not necessarily finite). The main tools are Kemer's representability theorem for PI algebras and Giambruno-Zaicev theorem on PI asymptotics. The proof uses the classification of finite simple groups.

In the lecture I will recall the necessary concepts and terminology. Joint work with Ofir David.

**Ann Kiefer**, Vrije Universiteit Brussel Coauthors: Eric Jespers, Ángel del Ro

# *Title:* A Presentation for a Group Acting Discontinuously on a Direct Product of Copies of Hyperbolic Spaces

The motivation behind this work is the investigation on the unit group of an order Oin a rational group ring QG of a finite group G. In particular we are interested in the unit group of ZG. Up to commensurability, this problem may be reduced to the study of the units of norm 1, denoted  $SL_n(O)$ , in some order O in each Wedderburn-Artin component of QG. Except for the so-called exceptional components, a specific finite set of generators can be given, up to finite index, for these groups. Exceptional components are simple components that are either non-commutative division algebras different from totally definite quaternion algebras or a  $2 \times 2$  matrix rings  $M_2(D)$ , where D is either Q, a quadratic imaginary extension of Q or a totally definite rational division algebra H(a, b, Q).

In some of these cases, the group  $SL_n(O)$  acts discontinuously on a direct product of several copies of hyperbolic 2- or 3-spaces. The aim is to generalize Poincaré's Polyhedron Theorem to direct products of hyperbolic spaces, and hence get a presentation for these groups. For the moment we have done this for the Hilbert Modular Group, which acts on  $H^2 \times H^2$ . Wei, Yangjiang, Guangxi Teachers Education University Coauthors: Tang, Gaohua and Nan, Jizhu

#### Title: The Iteration Digraphs of Group Rings

For a finite commutative ring R and a positive integer k > 1, we construct an iteration digraph G(R, k) whose vertex set is R and for which there is a directed edge from a to b if  $b = a^k$ . In this paper, we investigate the iteration digraphs  $G(F_{p^r}C_n, k)$  of  $F_{p^r}C_n$ , the group ring of a cyclic group  $C_n$  over a finite field  $F_{p^r}$ . We study the cycle structure of  $G(F_{p^r}C_n, k)$ , and explore the symmetric digraphs. Finally, we obtain necessary and sufficient conditions on  $F_{p^r}C_n$  and k such that  $G(F_{p^r}C_n, k)$  is semiregular.

Waldemar Holubowski, Inst. of Math, Silesian University of Technology

#### Title: Matrix representations of a free group

Matrix representations of abstract groups are popular due to computational advantages. In our talk we survey matrix representations of a free group in: SO(3, R), SL(2, Z) and UT(, Z) and show its applications

Yakov Karasik, Technion - Israel Institute of Technology Coauthors: Yuval Shpigelman (Technion - Israel Institute of Technology)

# *Title:* The Co-dmension Sequence of Finite Dimensional G Simple Graded Algebras

Precise knowledge of the identities of finite dimensional algebras in general seems to be a very hard task. Of course, knowing a set of generators of a given *T*-ideal would be a major step ahead but even then its not clear how to determine explicitly whether a polynomial is or is not generated by the given set. With this point of view it is natural (and many times more effective) to study general invariants attached to *T*-ideals of the free algebra. One of them (introduced by Regev in the 70s) is  $c_n(A)$  - the codimension sequence attached to the *T*-ideal of identities corresponding to an algebra *A*. In the 80s Regev, using results of Formanek, Procesi and Razmyslov in Invariants theory and Hilbert series, calculated asymptotically the codimension sequence of  $n \times n$  matrices over algebraically closed field of characteristic zero. Inspired by Regevs ideas, we pushed further his result and calculated asymptotically  $c_n^G(A)$  - the codimension sequence of matrix algebras *A* with elementary *G*-grading. Moreover, we used our calculation to prove that if *A* is a finite dimensional *G*-graded algebra then the polynomial part of  $c_n^G(A)$ 's asymptotics has degree  $(1 - \dim_F A)/2$  (this was conjectured by E. Aljadeff A. Giambruno and D. Haile).

In this talk I will present some of the ideas from Regevs work, and show how one can generalize them to tackle the elementary grading case. The lecture is based on a joint work with Yuval Shpigelman.

#### Gabriele Nebe, RWTH Aachen University

Coauthors: Oliver Braun, Renaud Coulangeon, Sebastian Schoennenbeck

#### *Title:* Computing unit groups of orders.

Let O be an order in some semisimple rational algebra A. Then its unit group  $O^{\times}$  is a finitely presented group. Together with Renaud Coulangeon and my two PhD students Oliver Braun and Sebastian Schnnenbeck we apply an algorithm developed by Voronoi in 1900 in the context of reduction theory of integral lattices to conpute generators and relations of  $O^{\times}$ . The additional data from Voronoi's algorithm can be used to solve the word problem in these generators.

Kenza Guenda, Faculty of Mathematics, University os Science and Technology USTHB, Algiers

#### Title: Ring's Application in Information Theory

Error correcting codes plays a fundamental role in the area of information theory. A special class of error correcting codes is the class of codes over rings. This class of codes found numerous applications in digital communications. A more recent novel application of codes over rings is in the area of DNA computing. This is a recent application of biology, rings theory and information theory which improves on conventional techniques in computation. There is also interest in the application of codes over rings for the development of quantum error correcting codes. Recently, the CSS construction have been extended to codes over rings. This produced numerous optimal codes taking advantage of the homogeneous weight. The purpose of this talk is to show and discuss those recent applications of codes over rings in the DNA computing, DNA modeling as well in the area of quantum information.

Hongdi Huang, Brock University Coauthors: Yuanlin Li

#### Title: On (5, 18) groups

A group G is said to have small squaring property on n-element subsets if for any nelement subset A of G, the number of the elements in  $A^2$  is no more than  $n^2$ , where  $A^2$ is the set of the product of any two elements in A. Furthermore, G is called a  $B_n$ -group if  $\#(A^2) < or = n(n+1)/2$ . Recently, Eddy and Parmenter generalized these notions to a new notion referred to as B(n,k) groups. A group G is called a B(n,k) group if #(A) = n implies  $\#(A^2) < \text{or} = k$  with k, or  $= n^2 - 1$ . Therefore, a  $B_n$ -group is a B(n, n(n+1)/2) group, and a group with small squaring property on n-element subsets is a  $B(n, n^2 - 1)$  group.

An interesting problem is to determine all B(n, k) groups. For any given k, G is obviously a B(n, k) group when #(G) < or = k, and such G is referred to as trivia. It is also easy to see that any abelian group G is a B(n, k) group when k > or = n(n+1)/2.

So what we are interested in is to determine all nonabelian nontrivial B(n,k) groups.

In this paper, we continue the investigation on B(5, k) groups for k = 18. We provide the complete characterizations of B(5, 18) non-2-groups and 2-groups in Section 2 and Section 3 respectively. Consequently, a complete characterization of B(5, 18) groups is obtained. It is shown that G is a B(5, 18) group if and only if one of the following statements holds: (1) G is abelian; (2) G is a trivial B(5, 18) group (i.e. #(G) <or = 18); (3)  $G < a, b, a^5 = b^4 = 1, a^b = a^{-1} >$ .

**Florian Eisele**, Vrije Universiteit Brussel Coauthors: A. Kiefer, I. Van Gelder

#### *Title:* Units of integral group rings of finite groups up to commensurability

Let G be a finite group. It has long been known that in most cases Bass units and bicyclic units generate a subgroup of finite index in U(ZG). The cases where this isn't true are characterized by the occurrence of certain  $2 \times 2$ -matrix rings in the Wedderburn decomposition of QG or an epimorphism from G onto a Frobenius complement. In this talk I will report on joint work with A. Kiefer and I. Van Gelder on the first of these two obstructions, the  $2 \times 2$  matrix rings. In this case a subgroup of U(QG) which is commensurable with U(ZG) can be described explicitly.

**Firdousi Begam**, Research Student, Department of Mathematics, Aligarh Muslim University (A.M.U), Aligarh-202002 Coauthors: Prof. Alveera Mehdi

#### Title: On Submodule Transitivity of QTAG-module

A right module M over an associative ring with unity is a QTAG-module if every finitely generated submodule of any homomorphic image of M is a direct sum of uniserial modules. Naji [2010] defined transitivity of modules with the help of U-sequences of the elements of M. M is transitive (fully transitive) if for any two elements  $x, y \in M$  with  $U(x) \subseteq U(y)$ , there exists an automorphism (endomorphism) f of M mapping x onto y. Different notions of transitivity were studied by Ayaz in 2012. Here we study submodule transitivity in the light of equivalent submodules. The submodules N, K of a QTAGmodule M are equivalent if there exists a height preserving automorphism f of M such that f(N) = K. There are examples when, for  $x, y \in M, xR$  is isomorphic to yR but M/xR is not isomorphic to M/yR. This motivates us to study submodule transitivity. We also investigate a class of modules  $\star$ -modules and it is found that  $\star$ -modules are strongly transitive with respect to countably generated isotype submodules. **Ofir David**, Technion - Israel Institute of Technology Coauthors: Eli Aljadeff (Technion - Israel Institute of Technology)

#### *Title:* Regular G-gradings on algebras

Let G be a finite abelian group. Regev and Seeman gave the following definition: A G-grading on an algebra A is called regular if

(1) for any  $a_g, b_h$  of degrees g, h respectively we have  $a_g b_h = q(g, h) b_h a_g$  where q(g, h) is a nonzero scalar.

(2) for every  $g_1, ..., g_n$  in G there are  $a_i$  in A of degrees  $g_i$  respectively such that  $a_1a_2...a_n! = 0$ .

Two examples for such regular gradings are the standard Z/2Z-grading on the infinite Grassmann algebra and the standard G-grading on a twisted group algebras. In this talk we present some properties of these algebras. In particular, we show that under some minimality conditions (defined by Regev and Bahturin), the size of the group G is an invariant of the algebra A. We do this by considering the (graded) polynomial identities of A, where the size - G - is an invariant of the ideal of polynomial identities of A.

Given time, we will show how this can be generalized to arbitrary finite groups (namely, not necessarily abelian).

#### Liang Shen, Southeast University

#### Title: A note on the Faith-Menal conjecture

A ring R is called right Johns if it is right noetherian and every right ideal of R is a right annihilator. R is called strongly right Johns if the  $n \times n$  matrix ring is right Johns for each positive integer n. The Faith-Menal conjecture is an open conjecture on QF rings. It says that every strongly right Johns ring is QF. It is proved that the conjecture is true if every closed left ideal of the ring is finitely generated. This result improves the known result that the conjecture is true if R is a left CS ring.

#### Marcin Mazur, Binghamton University Coauthors: R. Kravchenko and B. Petrenko

#### Title: Generators of maximal order

In this talk I will discuss the problem of finding the smallest number of generators of a maximal order in a semisimple algebra over a number field K as an algebra over the ring of integers of K. In an earlier joint work with Kravchenko and Petrenko, we have developed techniques to study generating sets of R-algebras which are finitely generated as R-modules. I will review some of the techniques and show how to apply them to get formulas for the smallest number of algebra generators of maximal orders. Our results extend many classical results obtained for maximal orders which are commutive.

**Jung Wook Lim**, Kyungpook National University Coauthors: R. Kravchenko and B. Petrenko

#### Title: S-Noetherian properties in a special pullback

Let A and B be commutative rings with identity,  $f : A \to B$  a ring homomorphism and J an ideal of B. Then the subring  $A \bowtie^f J := \{(a, f(a) + j) | a \in A \text{ and } j \in J\}$  of  $A \times B$  is called the amalgamation of A with B along with J with respect to f. In this talk, we investigate a general concept of the Noetherian property, called the S-Noetherian property which was introduced by Anderson and Dumitrescu, on the ring  $A \bowtie^f J$  for a multiplicative subset S of  $A \bowtie^f J$ . As particular cases of the amalgamation, we also devote to study the transfers of the S-Noetherian property to the constructions  $D + (X_1, ..., X_n)E[X_1, ..., X_n]$  and  $D + (X_1, ..., X_n)E[[X_1, ..., X_n]]$  and the Nagata's idealization.

This is a joint work with D.Y. Oh.

### Tuesday

Gurmeet K. Bakshi, Panjab University, Chandigarh, India Coauthors: Sugandha Maheshwary (Panjab University, Chandigarh, India)

#### *Title:* A complete irredundant set of strong Shoda pairs

Let QG be the rational group algebra of a strongly monomial group G. In order to understand the complete set of primitive central idempotents and the explicit Wedderburn decomposition of QG from the subgroup structure of G, an essential step is to determine a complete irredundant set of strong Shoda pairs of G. The study, in turn, helps to investigate the group of central units in the integral group ring ZG. This talk is a survey on this topic with some recent advances.

Yuri Bahturin, Dept. Math & Stat., Memorial University

#### *Title:* Relative growth of subsets in free groups and algebras

We discuss situations in groups and algebras where the numerical characteristics are given by infinite series rather than numbers. For examples, what happens to Schreier formula for the number of generators of a subgroup in a free group if the subgroup is of infinite index? Or how to draw distinction between two subalgebras of a free Lie algebra if they have the same rank? The talk is mostly based on joint results with Alexander Olshanskii.

**Eric Jespers**, Vrije Universiteit Brussel Coauthors: Jan Okninski

#### *Title:* Krull orders in nilpotent groups

For a submonoid S of a polycyclic-by-finite group G, it has been described when the semigroup algebra KS is a noetherian prime maximal order. For group algebras this is a result due to K. Brown. The obtained conditions show that such algebras can be characterized in terms of the monoid S. Concrete constructions include algebras of monoids of I-type that correspond to set theoretic solutions of the quantum Yang-Baxter equation. Chouinard has characterised the commutative monoid algebras KS that are Krull domains. This happens precisely when S is a Krull order in its (torsion-free) group of quotients and a description of such monoids is known.

A natural question is whether there exist finitely generated non-noetherian Krull domains K[S] with S a submonoid of a torsion-free nilpotent group G. In case G is of class two, a characterization is given of when KS is a Krull domain. Some examples are given that illustrate problems for a potential approach to higher nilpotency class.

This is joint work with. J. Okninski.

**David Riley**, Western University Coauthors: Eric Jespers

#### *Title:* Behaviour of the Frobenius map in a noncommutative world

The Frobenius map on a noncommutative associative unital algebra R over a field of prime characteristic p is given by exponentiation by p. In this talk, various properties of the Frobenius map will be discussed. We will start with the following natural question:

Problem 1. Precisely when is the Frobenius map (or some given power of the Frobenius map) a homomorphism on R when R is viewed as a ring?

It is a standard undergraduate exercise to show that this property holds whenever R is commutative (but not conversely). We will see that, if some power of the Frobenius map respects either addition or multiplication, then R satisfies an Engel identity, and, conversely, if R satisfies an Engel identity, then all sufficiently large powers of the Frobenius maps are ring homomorphisms with central image.

More general properties may be defined as follows: R is additively (respectively, multiplicatively) p-power-closed if, for every pair of elements x, y in R, the sum (respectively, product) of a common large p-power of x and y is a (single) p-power in R.

Problem 2. When is R additively (respectively, multiplicatively) p-power closed?

In the case when R is known to be a finitely generated PI-algebra such that each of its simple images has Schur index 1, we will see that R is additively (respectively, multiplicatively) p-power-closed if and only if R is Lie nilpotent. Since not all finite-dimensional perfect division algebras are Lie nilpotent, the Schur index condition cannot be omitted. A more complete answer to Problem 2 may prove difficult even in the following special case. Indeed, observe that R is trivially both additively and multiplicatively p-power-closed whenever the Frobenius map on R is surjective. In the latter situation, we call R perfect (as in the case when R is a field). We shall see that every finitely generated perfect PI-algebra is finite-dimensional (and therefore R is the direct product of finitely many finite-dimensional perfect division algebras over a perfect base field). This result might be viewed as an analogue of Kaplanskys famous partial solution of the Kurosh-Levitzki problem: every finitely generated algebraic PI-algebra is finite-dimensional. While Golod and Shafarevich were able to construct counterexamples to the Kurosh-Levitzki problem without the PI assumption, we believe the following problem remains open.

Problem 3. Is every finitely generated perfect algebra finite-dimensional? Specifically, is every finitely generated perfect nil algebra trivial?

# Geoffrey Janssens, Vrije Universiteit Brussel

Coauthors: Alexey Gordienko

#### *Title:* $ZS_n$ modules and polynomial identities with integer coefficients

In this talk I will speak about recent work joint with A. Gordienko, [1].

We study multilinear polynomial identities of unitary rings. In the same spirit as PI over fields we introduce integral codimensions.

We discuss the link with the classical codimension and calculate the exact codimensions of upper triangular matrices and the grassman algebra over a unitary ring.

We also prove that the study can be reduced to the study of proper polynomials (by showing an analog of young rule and of Drensky's theorem)

The talk will consist of some recalling of definitions, examples, motivation and finally by citing (not proving) the main theorems.

 $[1]ZS_n$ -modules and polynomial identities with integer coefficients (with A.Gordienko), Int. J. Algebra Comput. 23 (2013),no. 8, 1925-194

#### Allan Berele, DePaul University

#### *Title:* What are double centralizer theorems good for?

Schur's double centralizer theorem for the symmetric group and general linear group or Lie algebra has found important applications in representation theory, combinatorics, symmetric function theory, invariant theory and p.i. algebras. Since Schur's time there have been many other double centralizing theorems, and in many cases the applications have also been generalized.

Manizheh Nafari, DePaul University Coauthors: Michaela Vancliff

### *Title:* AS-Regular Graded Skew Clifford Algebras that are Twists of AS-Regular Graded Clifford Algebras

M. Artin, W. Schelter, J. Tate, and M. Van den Bergh introduced the notion of noncommutative regular algebras, and classified regular algebras of global dimension 3 on degree-one generators by using geometry (i.e., point schemes) in the late 1980s. They also defined twists by automorphisms and they proved that the regularity of algebras and GK-dimension are preserved under this twisting in the late 1980s. In 2010, T. Cassidy and M. Vancliff generalized the notion of a graded Clifford algebra and called it a graded skew Clifford algebra.

In this talk, We prove that if A is a regular graded skew Clifford algebra and is a twist of a regular graded Clifford algebra B by an automorphism, then the subalgebra of A generated by a certain normalizing sequence of homogeneous degree-two elements is a twist of a polynomial ring by an automorphism, and is a skew polynomial ring. We also present an example that demonstrates that this can fail when A is not a twist of B. This is the joint work with Michaela Vancliff.

**Jeffrey Bergen**, DePaul University, Chicago, IL, USA Coauthors: Piotr Grzeszczuk

# *Title:* Subrings of invariants of domains with finite Gelfand-Kirillov dimension.

In several papers with P. Grzeszczuk, we examine domains with finite Gelfand-Kirillov dimension and compare the dimension of the domain to the dimension of the invariants under the actions of derivations, skew derivations, and automorphisms. Our work is motivated by papers of J. Bell, A. Smoktunowicz, and L. Small.

#### Basil Nanayakkara, Brock University

# *Title:* Existence of terminal resolutions of geometric Brauer pairs in arbitrary dimension

A geometric Brauer pair is a pair (X, a) where X is a smooth quasi-projective variety over an algebraically closed field and a is an element in the 2-torsion part of the Brauer group of the function field of X. A geometric Brauer pair (Y, a) is a *terminal pair* if the Brauer discrepancy of (Y, a) is positive. We show that given a geometric Brauer pair (X, a), there is a terminal pair (Y, a) with a birational morphism  $Y \to X$ . In short, any geometric Brauer pair admits a terminal resolution.

**Jianlong Chen**, Southeast University Coauthors: Yanyan Gao, Yuanlin Li

#### *Title:* Some \*-clean group rings

A ring with involution \* is called \*-clean if each of its elements is the sum of a unit and a projection. It is obvious that \*-clean rings are clean. Vas asked whether there exists a clean ring with involution that is not \*-clean. In this paper, we investigate when a group ring RG is \*-clean, where \* is the classical involution on RG. We obtain necessary and sufficient conditions for RG to be \*-clean, where R is a commutative local ring and G is one of the groups C3, C4, S3 and Q8. As a consequence, we provide many examples of group rings which are clean but not \*-clean.

#### Chris Plyley, Western University

#### Title: On the Relationship of Actions and Gradings

The precise relationship between actions on an algebra and gradings is surprisingly intricate. Certain dualities have been long known to exist; for example, one classic duality states that if A is an algebra over a field K and G is a group isomorphic to its group of irreducible characters  $[\hat{G}]$ , then G acts by automorphisms on A precisely when A may be graded by  $[\hat{G}]$ . More generally, if H is a Hopf algebra acting on an algebra A, then A is called an H-algebra whenever its multiplication map  $A \otimes A \to A$  is an H-module homomorphism. In the case when H is finite-dimensional, semisimple, commutative, cocommutative, and splits over its base field, (for instance, when H=KG, the group (Hopf) algebra as above) it is known that A is an H-algebra precisely when the H-action on A induces a certain grading of A over a finite abelian group. In this talk we discuss how to extend these dualities to incorporate more general actions (for instance, actions by anti-automorphisms), and resultantly, more general types of gradings. Applications to polynomial identity theory are offered.

Tsunekazu Nishinaka, Okayama Shouka University

#### Title: Primitivity of group algebras of non-noetherian groups

A group of the class of non-noetherian groups which is, in particular, finitely generated has often non-abelian free subgroups; for instance, a free group, a locally free group, a free product, an amalgamated free product, an HNN-extension, a one relator group, a Fuchsian group etc (a free Burnside group is not the case, though).

In this talk, we focus on the following local property (\*) which is often satisfied by groups with non-abelian free subgroups. (\*): For each subset M of non-identity finite elements of G, there exists a subset X of three elements of G such that  $(x_1^{-1}g_1x_1)...(x_m^{-1}g_mx_m) = 1$  implies  $x_i = x_{i+1}$  for some i, where  $g_i \in M$  and  $x_i \in X$ . We can see that the group algebra KG of a group G over a field K is primitive provided G has a free subgroup with the same cardinality as G and satisfies (\*). In particular, for every countably infinite group G satisfying (\*), KG is primitive for any field K. As an application of this theorem, we can see primitivity of group algebras of many kinds of non-noetherian groups.

#### Stefan Catoiu, DePaul University, Chicago

#### Title: Pythagorean Counting in Rank One Restricted Enveloping Algebras

We investigate how the ancient Pythagorean counting of dots on a square or rectangular grid generalizes to dimension counting of sums of explicit indecomposable representations of the restricted enveloping algebra of  $sl_2$  and its quantum analogue.

## Thursday

**Jason Bell**, University of Waterloo Coauthors: Daniel Rogalski

#### *Title:* Free subalgebras of division rings

There has been a lot of work by Makar-Limanov, Shirvani, Goncalves, and others about the existence of free subalgebras, free groups, group algebras of free groups, and other free subobjects in a division ring D (or in its multiplicative group when talking about groups). We consider free subalgebras of a specific type of division ring. Let A be a countably generated noetherian domain over the complex numbers. Then one can form a quotient division algebra D=Q(A) by inverting the nonzero elements of A. We show that a dichotomy holds: D has a copy of the free algebra on two generators as a subalgebra unless D is finite dimensional over its centre, in which case it cannot have a free subalgebra on two generators. We give examples where this theorem applies.

**Ángel del Río**, Universidad de Murcia, Spain Coauthors: Osnel Broche

#### Title: On polynomials defining units in group rings

Let f be a polynomial in one variable with integral coefficients. Given a positive integer n, we say that f defines a unit on order n if f(x) is a unit in the integral group integral  $\mathbb{Z}\langle x \rangle$ , for x a group element of order n. We say that f defines generic units if there is a positive integer d such that f defines a unit on every order coprime with d. This is a concept introduced by Marciniak and Sehgal. Actually, they imposed that the polynomial f should be monic and characterized the monic polynomials defining generic units. We will prove that the assumption that a polynomial should be monic is irrelevant. More precisely we will prove that if f defines generic units then the leading coefficient of f is either 1 or -1 and as a consequence the polynomials defining generic units are those in Marciniak and Sehgal classification and their opposite.

Marciniak and Sehgal characterized also the polynomials of degree at most 3 defining a unit on some order. We will present some results on polynomials of degree 4.

#### Sergio R. López-Permouth, Ohio University

Coauthors: Jeremy Moore, Otterbein University. Nick Pilewski, Ohio University, and Steve Szabo, Eastern Kentucky University.

#### *Title:* Algebras having bases that consist solely of units

We consider algebras that have bases consisting entirely of units, called *invertible* algebras. Among other results, it is shown that all finite dimensional algebras over a field other than the binary field  $F_2$  have this property. Also, Invertible finite dimensional algebras over  $F_2$  are fully characterized. An earlier result that  $M_n(R)$  is an invertible R-algebra over an arbitrary ring R is extended here to show that if A is any R-algebra which is free as an R-module (and has a basis containing the element  $1 \in R$ ) then  $M_n(A)$  is an invertible R-algebra for any  $n \geq 2$ . Various families of algebras, including group rings and cross products, are characterized in terms of invertibility. In addition, invertibility of infinite dimensional algebras is explored and connections to the absence of the Invariant Basis Number (IBN) property are considered. (This talk is based on a paper by López-Permouth, Moore, Pilewski and Szabo.)

Jan Okninski, Institute of Mathematics, Warsaw University Coauthors: Eric Jespers

*Title:* Prime ideals in algebras determined by submonoids of nilpotent groups

The prime spectrum of the semigroup algebra K[S] of any submonoid S of a finitely generated nilpotent group is studied via the spectra of the monoid S and of the group algebra K[G] of the group G of fractions of S. It is shown that the classical Krull dimension of K[S] is equal to the Hirsch rank of the group G provided that G is nilpotent of class two. This uses the fact that prime ideals of S are completely prime. An infinite family of prime ideals of a submonoid of a free nilpotent group of class three which are not completely prime is constructed. They lead to prime ideals of the corresponding algebra. Prime ideals of the monoid of all upper triangular  $n \times n$  matrices with nonnegative integer entries are described and it follows that they are completely prime and finitely many.

Inneke Van Gelder, Vrije Universiteit Brussel Coauthors: Gabriela Olteanu

#### Title: Idempotents in group algebras over number fields

We describe the Wedderburn decomposition and the primitive central idempotents of a group algebra over a number field F of a finite strongly monomial group G. As an application, we study the number of simple components of FG and investigate when this number is minimal, i.e. when it coincide with the number of simple components of QG.

**Álvaro P. Raposo**, Universidad Politécnica de Madrid Coauthors: Paula Murgel Veloso

#### Title: Clean rings and group algebras. Part I

An *element* of a ring is *clean* if it is the sum of a unit and an idempotent. A *ring* is *clean* if every element in it is clean. The property of cleanness was formulated by Nicholson (1977) in the course of his study of exchange rings.

In this talk the property of cleanness as well as the basic properties of clean rings are introduced. The class of clean rings is located among other well known classes of rings. Then the focus is moved to group rings where these properties have been studied with the aim to characterize the rings R and groups G such that the group ring RG is clean. Starting in 2001 there have been several papers dealing with this questions and giving partial results: for R a semiperfect ring and G the group of order 2, for R a Boolean ring and G a locally finite group, for R a commutative ring and G a p-group, being p in the Jacobson radical of R, as well as some necessary conditions and some sufficient conditions for a group ring to be clean.

We have studied group algebras and tried to characterize the clean ones. In this first part of the talk we get address group algebras of finite groups. In the second part, delivered by Paula Murgel Veloso, we deal with some cases of infinite groups.

**Jairo Z Goncalves**, Department of Mathematics, University of Sao Paulo Coauthors: Donald S. Passman

# *Title:* Free groups in normal subgroups of the multiplicative group of a division rings

Let D be a division ring with center k and multiplicative group  $D \setminus \{0\} = D^{\bullet}$ , and let N be a normal subgroup of  $D^{\bullet}$ . We investigate various conditions under which N contains a free noncyclic subgroup F. In one of them, assuming that k is uncountable and  $N \supseteq G$ , a nonabelian solvable subgroup, we make use of a construction method due to Chiba to exhibit the free generators of F.

#### **Paula Murgel Veloso**, Universidade Federal Fluminense (UFF) Coauthors: Álvaro P. Raposo

#### Title: Clean rings and group algebras. Part II

Álvaro P. Raposo will present Part I of this subject, in which clean rings are defined, several properties of them are stated, and some instances of clean group rings are presented.

An *element* of a ring is *clean* if it is the sum of a unit and an idempotent. A *ring* is *clean* if every element in it is clean. The property of cleanness was formulated by Nicholson (1977) in the course of his study of exchange rings.

In the realm of group rings, these properties have been studied from 2001 on with the aim to characterize the rings R and groups G such that the group ring RG is clean. Several papers have been published in the field since then.

Our aim is to ultimately establish necessary and sufficient conditions for the group algebra KG to be a clean ring. In order to reach this goal, we have been following an "incremental" approach: we deal with group algebras over groups pertaining to certain classes, which grow more complex as we advance.

In this communication, we present a complete answer to this question in cases G is abelian, nilpotent, or supersolvable. We also present a partial result on for a class of groups that have a certain property that generalizes supersolvability.

#### Maya Van Campenhout, Vrije Universiteit Brussel

### *Title:* Finitely generated algebras defined by homogeneous quadratic monomial relations and their underlying monoids

We consider algebras over a field K with generators  $x_1, x_2, \ldots, x_n$  subject to  $\binom{n}{2}$  quadratic relations of the form  $x_i x_j = x_k x_l$  with  $(i, j) \neq (k, l)$  and, moreover, every monomial  $x_i x_j$  appears at most once in one of the defining relations. If these relations are non-degenerate then we showed that the algebra is left and right Noetherian, satisfies a polynomial identity and has Gelfand-Kirillov dimension at most n. In case the defining relations are square free this was already established by Gateva-Ivanova, Jespers and Okniński. To prove these results we investigated the structure of the underlying monoid S, defined by the same presentation. It is called a monoid of quadratic type.

#### Allen Herman, University of Regina

# *Title:* How to identify division algebras in the Wedderburn Decomposition of QG using GAP

I have recently developed GAP algorithms for calculating Schur indices that can be used to obtain a complete Wedderburn decomposition of a a rational group algebra QG. These algorithms have been included in releases of GAP's wedderga package as of January 2014. In this talk I will give a demonstration of how to use these algorithms and say a few words about their theoretical background.

### Friday

#### Benjamin Steinberg, City College of New York

#### *Title:* Etale groupoid algebras

An étale groupoid is a topological groupoid whose domain map is a local homeomorphism. Étale groupoid C\*-algebras play a fundamental role in noncommutative geometry. To each commutative ring with unit k and étale groupoid G with locally compact, totally disconnected unit space, I have associated a k-algebra with local units denoted kG. This construction simultaneously generalizes group algebras, inverse semigroup algebras, Leavitt path algebras, cross products of groups with idempotent-generated commutative k-algebras (when k is a field), and discrete groupoid algebras. We survey the theory including results on simplicity, semiprimitivity and primitivity, simple modules and Morita equivalence.

**Cesar Polcino Milies**, Instituto de Matemática e Estatística, Universidade de São Paulo

#### Title: Esential idempotents and Minimal Cyclic Codes

Let G be a finite group of order n,  $\mathbb{F}_q$  the field with q elements and assume that (n,q) = 1. Let e be an idempotent in  $\mathbb{F}_q$ . For a normal subgroup H of G set  $\hat{H} = 1/|H| \sum_{h \in H} h$ . If  $e\hat{H} = e$  then  $\mathbb{F}_q G.e \subset \mathbb{F}_q G\hat{H}$  and it is easy to see that, from the point of view of coding theory, this implies that the code defined by the ideal  $\mathbb{F}_q G.e$  is a repetition code.

A primitive idempotent of  $\mathbb{F}_q G$  is called *esential* if this does not happen; i.e. if  $e\hat{H} = 0$  for every normal subgroup H of G. This idempotents were first considered by Bakshi, Raka and A. Sharma in [1], where they are called non-degenerate.

If G is abelian, then G contains an essential idempotent if and only if G is cyclic. We shall give several algebraic characterizations of essential idempotents and compute their number in  $\mathbb{F}_q C_n$ , where  $C_n$  denotes the cyclic group of order n.

Using this concept, we are able to prove that every minimal abelian code is combinatorially equivalent to a minimal cyclic code.

Finally, if we denote by m the order of  $\overline{q}$  in  $\mathbb{Z}_n$ , set  $N = q^m - 1$  and  $\ell = N/n$  we establish a correspondence between essential idempotents of  $\mathbb{F}_q C_n$  and those of  $\mathbb{F}_q C_N$ .

These results are joint work with R. Ferraz and G. Chalom [2]

[1] G. K. Bakshi, M. Raka, A. Sharma, Idempotent Generators of Irreducible Cyclic Codes, *Proc. Int. Conf. Number Theory and Discrete Geometry*, Ramanujan Lecture Notes, 6, (2008), 13–18, ed. R. Balasubramanian, S. G. Dani, P. M. Gruber, R. J. Hans-Gill.

[2] G. Chalom, R. Ferraz e C. Polcino Milies, Essential Idempotents in Group Algebras and Minimal Cyclic Codes, *preprint* 

Hamid Usefi, Memorial University of Newfoundland Coauthors: Csaba Schneider and Iren Darijani.

#### Title: Classification of finite dimensional p-nilpotent restricted Lie algebras

I will talk about the classification of p-nilpotent restricted Lie algebras of finite dimension over a perfect field of characteristic p > 0. I will mention the classification of nilpotent Lie algebras of low dimension and the difficulties one might face trying to define and classify all possible p-maps on a given nilpotent Lie algebra.

**Gurmail Singh**, University of Regina Coauthors: Allen Herman

# *Title:* Lagrange type theorems for the torsion units of integral adjacency rings of finite association schemes.

An association scheme is a generalization of group. Using a generalization of Berman-Higman, I shall prove a Lagrange type theorem for the torsion units of integral adjacency rings of finite association schemes. Also I shall present a few other generalizations of results on torsion units of group rings to torsion units in the linear span of adjacency matrices associated with elements of schemes, over the field of complex numbers.

Daniel Smertnig, University of Graz, Austria

#### *Title:* Factorization theory in maximal orders in central simple algebras

Let R be a maximal order in a central simple algebra over a global field. Every non-unit of R that is not a zero-divisor can be expressed as a product of finitely many atoms (irreducible elements), but in general such a factorization is highly non-unique. Studying this non-uniqueness by means of arithmetical invariants has a long tradition in the commutative setting. In the present, noncommutative, setting we find that if every stably free left R-ideal is free, then key arithmetical invariants, such as sets of lengths and various catenary degrees, are finite and determined by combinatorial invariants of a suitable ray class group of the global field. This closely parallels the case for holomorphy rings in global fields. However, if R is a maximal order over a ring of algebraic integers and there exist stably free left R-ideals which are non-free, then many arithmetical invariants are infinite.

#### Mohamed A. Salim, UAE University, Al-Ain, U.A.E.

Coauthors: Victor Bovdi

#### Title: On the unit group of a commutative group ring

Let  $V(Z_{p^e}G)$  be the group of normalized units of the group algebra  $Z_{p^e}G$  of a finite abelian *p*-group *G* over the ring  $Z_{p^e}$  of residues modulo  $p^e$  with  $e \ge 1$ . The abelian *p*-group  $V(Z_{p^e}G)$  and the ring  $Z_{p^e}G$  are applicable in coding theory, cryptography and threshold logic (see [1], [4], [5] & [7]).

In the case when e = 1, the structure of  $V(Z_pG)$  has been studied by several authors (see [2]). The invariants and the basis of  $V(Z_pG)$  has been given by B. Sandling (see [6]). In general,  $V(Z_{p^e}G) = 1 + \omega(Z_{p^e}G)$ , where  $\omega(Z_{p^e}G)$  is the augmentation ideal of  $Z_{p^e}G$ . Clearly, if  $z \in \omega(Z_{p^e}G)$  and  $c \in G$  is of order p, then  $c + p^{e-1}z$  is a nontrivial unit of order p in  $Z_{p^e}G$ . We may raise the question whether the converse is true, namely does every  $u \in V(Z_{p^e}G)$  of order p have the form  $u = c + p^{e-1}z$ , where  $z \in \omega(Z_{p^e}G)$  and  $c \in G$  of order p?

We obtained a positive answer to this question and applied it for the description of the group  $V(Z_{p^e}G)$  (see [3]). Our research is a natural continuation of Sandling's results. References:

1. N.N. Aizenberg, A.A. Bovdi, E.I. Gergo, and F.E. Geche. Algebraic aspects of threshold logic, Cybernetics, 2:26–30, 1980.

2. A.A. Bovdi. The group of units of a group algebra of characteristic p, Publ. Math. Debrecen, 52(1-2):193-244, 1998.

3. V.A. Bovdi, M.A. Salim, On the unit group of a commutative group ring, Submitted for publication, p.1–10, 2012.

4. B. Hurley and T. Hurley, Group ring cryptography, Int. J. Pure Appl. Math, 69(1):67–86, 2011.

5. T. Hurley, Convolutional codes from units in matrix and group rings, Int. J. Pure Appl. Math, 50(3):431–463, 2009.

6. R. Sandling, Units in the modular group algebra of a finite abelian p-group, J. Pure Appl. Algebra, 33(3):337–346, 1984.

7. W. Willems, A note on self-dual group codes, IEEE Trans. Inform. Theory, 48(12):3107–3109, 2002.

**Sugandha Maheshwary**, Centre for Advanced Study in Mathematics Coauthors: Prof. Gurmeet K. Bakshi

# $\it Title:$ Wedderburn Decomposition of Rational group algebras - A computational approach

Let G be a finite group. We provide an algorithm to check whether G is normally monomial or not. We also develop an algorithm which yields strong shoda pairs of normally monomial groups and extremely strong shoda pairs of G. The efficiency of these algorithms has been illustrated with the computational comparison on a large sample of groups.

#### Mayada Shahada, Western University

Coauthors: Eric Jespers (Vrije Universiteit Brussel) David Riley (Western University)

#### *Title:* Multiplicatively Collapsing and Rewritable Algebras

A semigroup S is called n-collapsing, if for every  $a_1, ..., a_n$  in S, there exist functions  $f \neq g$  (depending on  $a_1, ..., a_n$ ) such that

$$a_{f(1)}...a_{f(n)} = a_{g(1)}...a_{g(n)};$$

it is called collapsing if it is n-collapsing, for some n. More specifically, S is called n-rewritable if f and g can be taken to be permutations; S is called rewritable if it is nrewritable for some n. Semple and Shalev extended Zelmanov's solution of the restricted Burnside problem by proving that every finitely generated residually finite collapsing group is virtually nilpotent. In this talk, we consider when the multiplicative semigroup of an associative algebra is collapsing. In particular, we prove the following conditions are equivalent, for all unital algebras A over an infinite field: the multiplicative semigroup of A is collapsing, A satisfies a multiplicative semigroup identity, and A satisfies an Engel identity. We deduce that, if the multiplicative semigroup of A is rewritable, then A must be commutative.

Andreas Bächle, Vrije Universiteit Brussel Coauthors: Leo Margolis

#### Title: On the prime graph question for 4-primary groups I

There will be two talks on this topic, the first being given by Andreas Bächle.

Let G be a finite group and V(ZG) the group of normalized units of the integral group ring ZG. The first Zassenhaus conjecture (ZC) states that every element of finite order of V(ZG) is conjugate to an element of G within the units of QG. This is a long standing and open conjecture and for non-solvable groups very few is known. If (ZC) is true, the so-called prime graph question has an affirmative answer:

(PQ): Let p, q be different primes. If V(ZG) has an element of order pq, does G also contain an element of order pq?

In the last years progress was made to answer this question for certain classes of groups. Among others, there is a positive answer to this question for all finite groups having an order divisible by at most 3 different primes.

We will report on current work to tackle this question for groups of an order involving at most 4 different primes and a new method developed to do so, which involves integral and modular representation theory. **Leo Margolis**, Universität Stuttgart Coauthors: Andreas Bächle

*Title:* On the prime graph question for 4-primary groups II See abstract above.

#### Wolfgang Kimmerle, University of Stuttgart

#### *Title:* Sylow properties of a finite group determined by its (group) rings

Let G be a finite group. The integral group ring of G is denoted by ZG and V(ZG) is the subgroup of the unit group U(ZG) consisting of all units with augmentation one. The question up to which extend torsion subgroups of V(ZG) are determined by G has been studied since G.Higman's thesis 1939 which completely settles the question when G is abelian or hamiltonian. More general one may ask in the sense of Richard Brauer's famous lectures on modern mathematics 1963 which properties of G are determined by its group algebras over fields or by its (ordinary) character table or by its representation rings.

The first part of the talk deals with the question whether Sylow like results are valid in V(ZG). In the second part similar questions are considered concerning character tables and specific representation rings.

#### References.

W. Kimmerle, Sylow like theorems for V (ZG), appears in Int. J.of Group Theory.
 W. Kimmerle, Unit Groups of Integral Group Rings: Old and New, Jahresber.
 DMV (2013) 115: 101 112.